Price Discrimination on Booking Time

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Abstract

Even if consumers are forward looking and free to choose when to purchase, a firm can price discriminate on booking time if consumers learn their valuations at different times and consumers who learn later have higher valuations. The model is related to our work on optimal screening with returns contracts Akan, Ata, and Dana [1], but here we consider a simpler binary-valuation distribution and consider more realistic consumer learning assumptions. The main contribution is to show that the profitability of screening on time is robust to relaxing the assumption that consumers learn instantaneously. In addition to analyzing a bad-news model in which information arrives gradually, we characterize a general bound on consumer optimism that guarantees the instantaneous learning results are robust.

1. Introduction

When can a firm price discriminate by varying its price over time for the same good? At one extreme it is well known that if consumers vary only in their valuations, firms cannot use time to price discriminate between them (i.e., Stokey [16]), while at the other extreme if consumers with different valuation purchase at exogenously different times, then firms can clearly discriminate between them.

We show that price discrimination on booking time is feasible if the firm would like to charge a higher price to consumers who learn later. This paper is closely related to Akan, Ata and Dana [1] in which we analyze optimal pricing when the firm can write contracts with consumers at the start of the game, before consumers have learned their valuations. The optimal contracts are returns contracts, or options contracts, and stipulate both an upfront price and a partial refund if the consumer later opts not to consume the good. Both payments vary with the deadline by which the consumer must exercise the refund option. While interesting and relevant, the variation in refund contracts predicted by that paper is seen only a few industries.

In contrast, this paper considers a much simpler binary distribution for consumer’s valuations. This assumption has several advantages. First, we show the optimal pricing policy can often be implemented with simple spot prices. Second, the optimal contracts are more intuitive and hence have pedagogical value. The distortions in the size of the return that we describe in our related paper are interesting and important, but make some of the basic insights about price discrimination on booking time less obvious. Third, we are able to characterize optimal second-best prices for a wider variety of consumer heterogeneity: in our related paper, we consider second-best pricing only when the distributions can be ranked by first order stochastic dominance. And finally, the binary distribution assumption allows us to make much less restrictive assumptions on consumer learning. In our related paper we assume consumers learn

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their valuations instantaneously, while in this paper we consider a wide variety of information structures and verify that our basic insights are robust to the assumption of more realistic information structures.

Not surprisingly, booking time is a powerful screening device when consumers learn their valuations at different times and their valuations are correlated with when they learn. When consumers who learn their valuations later have higher valuations (conditional on wanting the good) then using spot prices that increase over time is profitable and may even extract all of the consumer surplus. Consumers who learn their valuations late are deterred from purchasing earlier because doing so requires them to commit before they know if they want the good, and consumers who learn their valuations early are deterred from purchasing later because the price is higher. This result may not seem surprising since it is what we would expect if consumers were arriving at different times and consumers who arrived later had higher valuations, but we think an important contribution of our work is understanding learning as a possible foundation for the heterogeneous-arrival-time assumption frequently made in other revenue management and dynamic mechanism design models.

Our work contributes to the literature on time-varying prices, or intertemporal price discrimination. Much of this work is in the durable-good setting in which consumers buy only once and prefer to consume sooner (e.g., a durable such as a car, or movie which is consumed only once) and these papers generally assume the firm can commit to its pricing policy. Stokey [16] shows time doesn’t help firms discriminate when consumers vary only in their valuations, but if consumers are myopic, then declining prices can discriminate between them (and more generally when consumers have a higher discount rate than the firm, see Landsberger and Meilijson [10]. Su [17] shows declining prices are also optimal when consumers with higher valuations have higher waiting costs or are more impatient. Board [3] shows intertemporal price discrimination is profitable when consumers valuations are correlated with when they appear in the market. Sobel [15] and Conlisk, Gerstner, and Sobel [4] find that even when consumers vary only in their valuations, time-varying prices are profitable if consumers vary in when they arrive (even if valuations and arrivals are uncorrelated). The firm sets a high price until the unmet demand of consumers with valuations below the high price gets sufficiently large, and the firm temporarily drops price and clears the backlogged demand. Öry [12] considers a closely related model of Internet advertising in which sellers can recall buyers and argues that advertising costs increase profits when the commitment assumption is relaxed.

Some of the intuition is similar in advance-purchase and revenue management settings in which consumers are booking a reservation for future consumption (such as a sporting event or airline travel), but discounting is not relevant in these settings, so the standard exceptions to [16] don’t hold. But clearly firms could intertemporally price discriminate with declining prices in both settings if consumers had an attention or search cost that was correlated with their valuation. What is common to both settings is that price discrimination arises because of other heterogeneity in consumers that is correlated with the heterogeneity in their valuations. So increasing prices will be profitable if consumers who arrive later have higher valuations, or consumers who learn later have higher valuations.

Several other papers have modeled advance-purchase pricing when consumers are learning their valuations over time. Courty [5] considers the decision to sell to consumers either before or after they have learned their valuations. He shows there is a tradeoff between allocative efficiency (which favors selling after) and reducing consumer heterogeneity (which typically favors selling before). Dana [7] analyzes a competitive market with uncertain demand and fixed capacity and shows that in equilibrium some firms set lower prices and sell to consumers who purchase in advance before they learn while other firms sell to consumers who wait to purchase until after they learn. Gale and Holmes [8] analyze a related monopoly model in which the firm price discriminates by selling to some consumers before they learn their valuations and others afterwards. Nocke and Peitz [11] use a mechanism design framework to analyze optimal advance purchase pricing when consumers vary in how much information they have ex ante. Courty and Li [6] consider optimal screening when the firm can sell returns contracts to all consumers ex ante and vary the size of the refund in order to screen consumers. They show it is optimal to distort the size of the refund to some consumers in order to extract more surplus from others. However, this paper, and [1] are the only ones to look at learning when consumers realized valuations are systematically related to when they learn.

Advance-purchase pricing has also been widely studied in operations management literature, though that literature largely considers a stochastic demand environment and capacity constraints. Gallego and Şahin [9], and other recent work in operations management, models consumers as forward-looking and strategic. An excellent survey of this literature is Talluri and van Ryzin [18] (see also Shen and Su [14]).

Our work is also related to many other papers in the advance-purchase pricing literature, and to many recent papers in dynamic mechanism design including several papers that model consumer arrivals. For a more complete discussion
of the literature see [1] and two excellent recent surveys, Vohra [19] and Bergemann and Said [2].

2. The Basic Model

A single, risk-neutral firm sells a homogenous good with a unit cost, \( c \), to heterogeneous, risk-neutral, privately-informed consumers. Consumers are privately informed about when they will learn their valuations and about the distribution of their valuations for the good. The firm chooses a direct-revelation mechanism to maximize its expected profits.

Each consumer is privately informed about her type, \( t \in [0, T] \), and later learns her valuation, \( v \), which also becomes private information.\(^1\) The type \( t \) represents the time at which she learns her valuation, but \( t \) is also a parameter in the distribution of her valuation, \( v \). The function \( f(t) \) is the number of consumers of each type where \( \int_0^T f(t) dt = 1 \).

We assume the valuation, \( v \), has a simple binary distribution: type \( t \)’s valuation is high and equal to \( V(t) > 0 \) with probability \( \pi(t) > 0 \) and is low and equal to 0 with probability \( 1 - \pi(t) > 0 \). We also assume for simplicity that \( V(t) > c \), for all \( t \), so trade is always efficient when consumers learn they have a high valuation. And we assume that all consumption takes place in the future, after consumers learn their valuations, so discounting can be ignored.

Each consumer maximizes her consumer surplus, her valuation \( v \) if she receives the good and 0 otherwise less the payment to the firm. Under our assumptions a consumer’s expected valuation, \( \pi(t)V(t) \), will typically be correlated with \( t \), the time at which she learns her valuation. So it may be profitable for the firm to price discriminate using booking time as a screening device.

As a base case, consider optimal pricing with complete information. In this case the firm can extract all of the consumer surplus by setting an ex post price \( p(t) = V(t) \). That is, when the firm can observe each consumer’s type and charge each consumer a different price, then the firm can capture all of the consumer surplus.

3. First-Best Pricing

When consumers’ types are private information, or for some other reason the firm cannot directly price discriminate, the firm may still be able to indirectly price discriminate. In this section we show that when consumers learn their valuations at different times, it may still be possible for the firm to extract all the surplus by screening on when consumers make their purchase decisions.

Intuitively, the firm can charge sequential spot prices \( p(t) = V(t) \) under two conditions. First, it must be true that no consumer prefers to wait and purchase later, which is true as long as \( V(t) \) is nondecreasing so the price is nondecreasing. And second, it must be true that no consumer prefers to purchase earlier, which is true as long as \( V(t') \geq \pi(t)V(t) \), for all \( t, t' \) such that \( t' < t \), so that the payment \( V(t') \), which is made even when the consumer’s valuation turns out to be low, exceeds the expected valuation, \( \pi(t)V(t) \). This is shown in Proposition 1.

**Proposition 1.** The firm can implement the complete-information, or first-best, outcome with a sequence of increasing, non-refundable prices \( p(t) = V(t) \) if and only if (i) \( V(t) \) is nondecreasing, and (ii) \( V(t') \geq \pi(t)V(t) \), for all \( t, t' \) such that \( t' < t \), or equivalently \( V(0) \geq \pi(t)V(t) \), for all \( t \).

**Proof.** Clearly a type \( t \) consumer purchases at price \( p(t) = V(t) \) at time \( t \) if her valuation is equal to \( V(t) \) and does not purchase if her valuation is 0. So consumers are willing to purchase when their valuation is high and not otherwise.

Incentive compatibility also requires a type \( t \) consumer prefers to purchase at time \( t \) and not \( t' \). If a type \( t \) consumer imitates a type \( t' > t \) consumer and purchases at time \( t' \), then her consumer surplus is \( v - p(t') = v - V(t') \), which is negative even if her valuation is high (equal to \( V(t) \)) because \( V(t') \) is nondecreasing in \( t \). If a type \( t \) consumer imitates a type \( t' < t \) consumer and purchases at time \( t' \), then her expected consumer surplus is \( \pi(t)V(t) - p(t') = \pi(t)V(t) - V(t') \) which is negative since \( V(t') \geq V(0) \) by condition (i), and \( V(0) \geq \pi(t)V(t) \) by condition (ii).

\(^1\)Formally, each consumer’s type is a pair, \((t, v)\), however, given that whether or not she wants the good is completely informative about her valuation, we find it simpler to refer to her type as \( t \).
Condition (i) is clearly necessary since otherwise some consumer would be able to pay less by waiting and buying at time $t' > t$. And Condition (ii) is clearly necessary since otherwise there exists a type $t$ consumer who could earn strictly positive surplus by imitating a type 0 consumer.

Finally, condition (i) can be used to simplify condition (ii), so Proposition 1 implies that the first best can be implemented with spot prices if and only if (i) $V(t)$ is nondecreasing, and (ii) $V(0) \geq \pi(t)V(t)$, for all $t$. ■

Notice that the necessary and sufficient conditions are different from the necessary and sufficient conditions for achieving the first-best in [1]. In that paper the firm can implement the first best if and only if $E_t[\max(V(t') - c, 0)]$ is greater than $E_t[\max(V(t) - c, 0)]$ for all $t' > t$ (increasing expected gains from trade, or IEGT) and also that $E_t[\max(V(t') - c, 0)] \geq E_t[V(t)] - c$ for all $t' < t$ (sufficient option value, or SOV). Condition IEGT implies that a consumer of type $t$ does not want to imitate a consumer of type $t' > t$, since they pay a higher expected price, and Condition SOV implies that a consumer of type $t$ is not willing to imitate a consumer of type $t'$, since the price they pay, $E_t[V(t) - c] + c$, includes an option to return the good at time $t'$, and type $t$ will never exercise that return option, so the price exceeds their valuation.

The difference in the necessary and sufficient conditions is because the distributional assumptions of the two papers are different, and because implementation with spot prices (which are equivalent to fully refundable purchases) is more restrictive than implementation with arbitrary refund contracts. In [1], we assume the distribution has full support so only returns contracts with a return price of $c$ can implement the first best, though the firm can offer any return contract. Under the binary distribution assumption in this paper the distribution of $v$ is a single point, $V(t)$, after conditioning on $v > c$, so the firm is able to extract all of the consumer surplus with spot prices (which are equivalent to fully refundable tickets) without introducing ex post distortions. This increases the value of the option to return (or equivalently the option to decline to purchase) and makes imitating a lower type even more costly. The broader implication is that the second set of conditions (the downward incentive constraints) are easier to satisfy here than in [1].

The first set of conditions (the upward incentive constraints, condition (i) and IEGT) are also different. In particular, note that if $V(t)$ is decreasing but $\pi(t)[V(t) - c]$ is increasing, then the firm cannot implement the first best with sequential spot prices, but, as long as SOV holds, the firm can implement the first best by charging each consumer $\pi(t)[V(t) - c] + c$ with an option to return for a payment of $c$ at time $t$. Type $t$ does not want to imitate type $t' > t$ because the expected payment is larger. So under the binary distribution assumption, it is possible that the first-best can be implemented, but not with sequential spot prices.

Put more succinctly, IEGT and SOV are sufficient for the first-best to be implemented with a menu of returns contracts. This follows from Proposition 1 in [1]. Moreover, this implies that if $\pi(t)[V(t) - c]$ is increasing, $\pi(t')(V(t') - c) + c \geq \pi(t)V(t)$ for all $t' < t$, and $V(t)$ is decreasing, then the firm can implement the first best, but cannot do so with spot prices. Since IEGT and SOV hold, the first-best is feasible using contracts with a refund price equal to $c$, but condition (i) fails to hold so spot prices don’t implement the first best.

Note also that if $V(t)$ is increasing, but $E[V(t) - c] = \pi(t)[V(t) - c]$ is decreasing, the firm may still be able to implement the first-best under the binary distribution. In particular, as long as condition (ii) holds, then the firm can still implement the first-best. Again, this is because under the binary assumption sequential spot prices are not distortionary, so implementing the first-best is often possible when it isn’t for more general distributions.

4. Second-Best Pricing

While we don’t fully analyze the optimal prices for all $\pi(t)$ and $V(t)$, the binary distribution assumption allows us to do so much more generally than in our companion paper, [1].

For simplicity we now assume that the firm finds it profitable to serve every high-valuation consumer. This is equivalent to imposing an ex ante individual rationality constraint for all types. This assumption was unnecessary in the discussion of first-best pricing since selling to all types clearly maximizes profits. It implies that if the firm were constrained to set a single price, it would charge $\min_t V(t)$. Relaxing this assumption is formally simple, but notationally complex.

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2SOV holds if $E_t[V(t') - c] + c \geq E_t[V(t)]$ for all $t' < t$, or for a binary distribution if $\pi(t')(V(t') - c) + c \geq \pi(t)V(t)$ for all $t' < t$, which implies $V(t') \geq \pi(t)V(t)$, for all $t' < t$, so SOV implies condition (iii).
The following lemma helps us restrict our attention to mechanisms that can be implemented with a sequence of spot prices. The proof is in the Appendix.

Lemma 1. If \( \pi(t) \) is weakly decreasing, then selling using a sequence of spot prices is as profitable as any other selling mechanism.

Intuitively, when \( \pi(t) \) is increasing, then \( E[v] = \pi(t)V(t) \) can be increasing even when \( V(t) \) is decreasing or constant. In this case options contracts can extract more surplus than spot prices because they consumers who learn late are willing to pay more up front for the option to choose later.\(^3\) When \( \pi(t) \) is decreasing, even if \( \pi(t)V(t) \) is increasing, the surplus can be extracted ex post through higher spot prices.

Note that the assumption on \( \pi(t) \) is a sufficient and not necessary condition. Conditions (i) and (ii) of Proposition 1 can hold when \( \pi(t) \) is increasing, in which case not only are sequential spot prices optimal, but they achieve the complete-information outcome. However, clearly if \( \pi(t) \) increases too quickly, or \( \pi(t) \) is too large, then condition (ii) of Proposition 1 will be violated.

Going forward, we will focus on spot prices, so we will assume \( \pi(t) \) is weakly decreasing. An alternative assumption is that the firm can make non-refundable (or fully refundable) sales. A spot price at time \( t \) is a non-refundable sale offered only at time \( t \). Equivalently, a spot price at time \( t \) can be viewed as a fully refundable sale anytime before time \( t \) that is only refundable until time \( t \). Thus options are not allowed (upfront payments for the right to purchase later at a given price, or equivalently up front payments with partial refunds for returns). This restriction might be reasonable because of transactions costs.

Using Lemma 1, we now characterize the optimal prices when \( \pi(t) \) is weakly decreasing. The firm’s problem is to chooses its prices, \( p(t) \), to maximize its profits

\[
\max_{p(t)} \int_0^T p(t)\pi(t) f(t) dt
\]

subject to

\[
p(t) \leq V(t) \quad \text{(i)}
\]

\[
p(t) \leq p(t')/\pi(t) \text{ for all } t' < t, \text{ and} \quad \text{(ii)}
\]

\[
p(t) \leq p(t') \text{ for all } t' > t. \quad \text{(iii)}
\]

The first constraint is the individual rationality constraint (recall we assumed the firm finds it profitable to sell to every high-valuation consumer). The second constraint is an incentive compatibility constraint. Consumer surplus for type \( t \) if she waits until time \( t \) to make her purchase decision is \( \pi(t)(V(t) - p(t)) \) (she purchases with probability \( \pi(t) \)), and if she imitates type \( t' < t \) by purchasing at time \( t' \), her consumer surplus is equal to \( \pi(t)V(t) - p(t') \) (she always purchases, but receives a positive value with probability \( \pi(t) \)). The former is larger than the latter if \( p(t) \leq p(t')/\pi(t) \). And the third constraint is the incentive compatibility constraint for type \( t \) with respect to type \( t' > t \). This decision is made after type \( t \) learns her valuation, so the constraint just requires that price be non-decreasing.

The following proposition describes the optimal pricing policy under a very mild restrictive condition on the function \( V(t) \).

Proposition 2. Suppose \( V(t) \) is continuously differentiable and the interval \( [0,T] \) can be partitioned into a finite number of subintervals in which \( V(t) \) is weakly increasing or weakly decreasing. Then if \( \pi(t) \) is decreasing it follows that the firm’s optimal pricing policy is the weakly increasing sequence of spot prices equal to

\[
\hat{p}(t) = \min_{t' \geq t} \hat{p}(t'),
\]

where \( \hat{p}(t) = \min \{V(t), V_{\min}/\pi(t)\} \) and \( V_{\min} = \min_{t \geq 0} V(t) \).

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\(^3\)When \( \pi(t) \) is increasing, \( V(t) \) is decreasing, and the product \( \pi(t)V(t) \) is increasing, the best the firm can do with sequential spot prices is set \( p(t) = V(T) \). But the firm does better by charging \( \pi(t)V(t) - c \) up-front to every consumer and then charging them \( c \) to get the good at time \( t \), if they want it. As long as \( \pi(t)V(t) \) isn’t increasing too fast these prices are incentive compatible.
increasing and type probability that she has a high valuation, which will change over time as information arrives. In our basic model, a instantaneously at time $t$, imitate a type 0 consumer, in which case she will pay $p$ from constraint (iii) that $p = V_{\min}$ for all $t \in [0, t_{\min}]$ (and if $t_{\min}$ is not uniquely defined, then this equality holds for all $t_{\min}$). So if a consumer of type $t$ wants to imitate a consumer of type $t' < t$, then without loss of generality she will imitate a type 0 consumer, in which case she will pay $V_{\min}$ for the good. It follows that constraint (ii) can be written $p(t) \leq V_{\min}/\pi(t)$, $\forall t$.

Clearly $\hat{p}(t)$ maximizes the firm’s profits subject to only constraints (i) and (ii). However, $\hat{p}(t)$ may not be increasing, so $\hat{p}(t)$ may not satisfy constraint (iii).

Note that constraint (iii) does not bind at $t = T$, so the solution clearly satisfies $\hat{p}(T) = \hat{p}(T)$, which means the solution is given by (2) for $t = T$. We now show the rest of the solution satisfies (2) by recursion. Let $t_i, i = 1, \ldots, n$ be the set of points in $[t_{\min}, T]$ which define the intervals on which $V(t)$ is weakly increasing or weakly decreasing. Suppose that the solution $\tilde{p}(t)$ is known for some (possibly degenerate) interval $[\tau, T]$ where $\tau \in T = \{t_{\min}\} \cup \{t_1, \ldots, t_n\} \cup \{T\}$. Let $\tau'$ be the largest element in $T$ that is less than $\tau$, so either $V'(t) \geq 0$ or $V'(t) \leq 0$ for all $t \in [\tau', \tau]$.

If $V'(t) \geq 0$ for all $t \in [\tau', \tau]$, then constraint (iii) does not bind, and $\hat{p}(t)$ is weakly increasing for all $t \in [\tau', \tau]$. This implies $\tilde{p}(t) = \hat{p}(t)$ for all $t \in [\tau', \tau]$, and the solution satisfies (2) for all $t \in [\tau', \tau]$.

If $V'(t) \leq 0$ for all $t \in [\tau', \tau]$, then constraint (iii) binds so the solution satisfies $\tilde{p}(t) = \hat{r}(t)$ for all $t \in [\tau', \tau]$, which clearly satisfies (2).

So we have proven that the solution satisfies (2) for all $t \geq \tau'$. Proving the solution satisfies (2) for all $t \geq t_{\min}$, and hence for all $t \in [0, T]$ can now be done recursively by going backwards through the intervals defined by the elements of $T$.

Note that Proposition 2 implies that the price is constant and equal to $V_{\min}$ if both $V(t)$ and $\pi(t)$ are decreasing.

We have now characterized the optimal prices when $\pi(t)$ is decreasing (Proposition 2), and also when $V(t)$ is increasing and $V(0) \geq \pi(t)V(t)$ for all $t$ (Proposition 1). Characterizing the optimal pricing policy more generally is feasible, but since sequential spot prices are not optimal it is less clear that this analysis adds anything not already in [1].

5. Consumer Learning

A strong assumption made in the above model, and also made in [1], is that type $t$ consumers learn their valuations instantaneously at time $t$. A natural question is whether the perfect discrimination result is robust to relaxations of this assumption. In this section we extend our analysis by considering a richer set of information structures for consumers.

In our simple binary distribution model, a consumer’s beliefs can be represented by a single real number, the probability that she has a high valuation, which will change over time as information arrives. In our basic model, a type $t$ consumer begins at time 0 with the prior that the probability of her having a high valuation, $V(t)$, is $\pi(t)$ and the probability of her having a low valuation, 0, is $1 - \pi(t)$. This belief doesn’t change until time $t$ when the consumer learns her valuation. After time $t$ a fraction $1 - \pi(t)$ of the type $t$ consumers believe their valuation is 0 with probability 1, and a fraction $\pi(t)$ of type $t$ consumers believe their valuation is $V(t)$ with probability 1.

In order to model learning more generally, we need to define the consumer type space more generally. We assume new information arrives over time, and that consumers update their beliefs in response to that information. We make the simplifying assumption that at time $t' < t$ a consumer’s type can be represented as a pair $(t, \rho(t'))$ where $t$ obviously represents the consumer’s initial type and $\rho$ represents the posterior probability that $v = V(t)$ at time $t'$. In other words, $\rho$ is a martingale, a sequence of random variables in which the expected value of the next value (and all future values) is equal to the current value in the sequence. This assumption implies that all of the information that a type $t$ consumer has acquired between time 0 and time $t'$ (other than their type $t$) can all be summarized by the posterior $\rho$. Naturally, we assume that at time 0 all type $t$ consumers are of type $(t, \pi(t))$. That is, they have a common prior $\pi(t)$, which is consistent with our basic model. And we assume that at time $t$ all type $t$ consumers are either of type $(t, 0)$ or type $(t, 1)$. That is, by time $t$ they know with certainty whether or not $v = V(t)$, which is again consistent with our basic model.

So we can represent the information structures for a type $t$ consumer at time $t'$ as a distribution function over $\rho$. Let $F(\rho'; t)$ denote cumulative distribution function over $\rho(t') \in [0, 1]$. So $F(\rho'; t)$ is the likelihood that a type $t$ consumer at time $t'$ believes her valuation is high with probability $\rho$ or less. In our basic model with instantaneous
learning, the distribution of beliefs, \( F(\rho'|t'; t) \), has all of the mass at \( \pi(t) \) whenever \( t' < t \), so \( F(\rho'|t'; t) = 0 \) for \( \rho(t') < \pi(t) \) and \( t' < t \) and \( F(\rho'|t'; t) = 1 \) for \( \rho(t') \geq \pi(t) \) and \( t' < t \), but at time \( t' \geq t \), the mass is divided between 0 and 1, so \( F(\rho'|t'; t) = \pi(t) \) for \( \rho(t') < 1 \) and \( t' \geq t \) and \( F(\rho'|t'; t) = 1 \) for \( \rho(t') = 1 \) and \( t' \geq t \).

In our generalization of the basic model, the key characteristics of a type \( t \) consumer are unchanged – the extensive distribution of her valuations is the same, and we assume that she knows her valuation by time \( t \) – however, we generalize the information structure between time 0 and time \( t \). The CDF \( F(\rho'|t'; t) \) represents the distribution of possible beliefs a type \( t \) consumer can have at time \( t' \) (this distribution can be thought of as the consumer’s prior at time 0 about what they might believe at time \( t' \), but more importantly as the firm’s beliefs at time \( t' \) since we assume the firm knows what information the consumer might have acquired, but not the realization of that information.)

A critical characteristic of \( F(\rho'|t'; t) \) for our analysis is the belief of the most optimistic consumer, or the lowest value of \( \rho \) for which \( F(\rho'|t'; t) = 1 \). Let \( \phi(t'; t) \) denote this minimum value, that is the value of \( \rho \) such that every type \( t \) consumer has beliefs that are less optimistic than \( \rho \) at time \( t' \).

Clearly if a type \( t \) consumer learns that her valuation is high with probability one at any time \( t' < t \), in which case \( \phi(t'; t) = 1 \), then that consumer will purchase immediately (if she hasn’t already done so). However, perfect discrimination may still be feasible if the beliefs of the most optimistic consumer are sufficiently pessimistic, or \( \phi(t'; t) \) doesn’t exceed some threshold.

What is important is not whether consumers learn gradually or instantaneously, but whether the most optimistic of consumers remains sufficiently pessimistic. This is easy to satisfy by assuming consumers learn instantaneously, but can also be satisfied when consumers learn gradually. We formalize this in the following proposition and then consider a specific example which satisfies the sufficient conditions.

**Proposition 3.** The firm can implement the complete-information outcome, and extract all of the surplus from consumers, with a sequence of increasing, non-refundable prices \( p(t) = V(t) \) if and only if (i) \( V(0) \geq \pi(t)V(t) \), for all \( t \), and (ii) \( V(t') \leq V(t)/V(t) \) for all \( t' < t \).

**Proof.** Clearly the complete-information prices are not incentive compatible unless (iii) holds, and are incentive compatible if (i), (ii), and (iii) holds. The rest follows from Proposition 1. ■

Our analysis does not specify the learning dynamics, but instead assumes that they can represented at any point in time by a distribution of posteriors. So clearly we are allowing for a rich variety of learning assumptions. The critical property that preserves incentive compatibility is the upper bound on consumer optimism, not how quickly or slowly consumers learn. So our basic model is likely to be robust to a variety of relaxations in which consumers learn slowly rather than instantaneously. Below we consider one specific learning model that satisfies this property.

Note that Proposition 3 can be generalized, but the prices are incentive compatible if \( \phi(t'; t)p(t) \leq p(t') \), which isn’t necessarily implied by \( \phi(t'; t) \leq V(t')/V(t) \) for all \( t' < t \).

Finally, an important observation is that at any time \( t \) a consumer’s type is multidimensional. At time 0 consumers types are unidimensional and give by \( t \). But afterward time 0, in addition to knowing the posterior probability that they will have a high valuation, consumers also know the time \( t \) at which their posterior will converge to 0 or 1. In other words, at time \( t' \) two consumers who were initially types \( t_1 > t \) and \( t_2 > t \) could have the same posterior probabilities, so the same distribution over whether their valuations are high or low, but have different expectations about when they will learn their valuations. They could even have the same distribution of valuations if \( V(t_1) = V(t_2) \). So our information assumptions violate the assumptions in Pavan, Segal and Toikka [13]. This is important because they show that first best is not feasible under their assumptions in a broad class of dynamic mechanism design problems. In [1] the consumer’s type is one dimensional, but instantaneous learning violates the bounded impulse response function assumption in [13].

**Example**

The following “bad-news” example demonstrates that condition (iii) holds for reasonable assumptions on the information structure. Suppose that a type \( t \) consumer starts at time 0 with the prior that her valuation is \( V(t) \) with probability \( \pi(t) \) and then may receive a signal between time 0 and time \( t \) that reveals that her true valuation is 0. The probability density, \( h(t') \), of receiving the signal is uniformly distributed on the time interval \([0, t] \), so \( h(t') = (1 - \pi(t))/t \)
for \( t' \in [0, t] \), and the associated cumulative distribution function is \( H(t') = (1 - \pi(t)) \frac{t'}{t} \), which clearly approaches 1 - \( \pi(t) \) as \( t' \) approaches \( t \).

In this example we show that condition (iii) of Proposition 4 is satisfied as long as \( V''(t) \leq 0 \).

Between time 0 and time \( t \) the consumer receive a signal informing her that her valuation is zero, but if she does not receive a signal, she will update her beliefs and expect to have a high valuation with greater and greater probability. At time \( t' \), conditional on not having already received a signal revealing that her valuation is 0 between time 0 and time \( t' \), a type \( t \) consumer believes she has a high valuation is \( \pi(t)/(1 - F(t')) \), or

\[
\phi(t'; t) = \frac{\pi(t)}{1 - (1 - \pi(t))t'}
\]

for all \( t' \in [0, t] \). This is just the unconditional probability of a high valuation divided by the unconditional probability of receiving no bad news by time \( t' \), which approaches 1 as \( t' \) approaches \( t \). This information structure satisfies condition (iii) of Proposition 4 if the consumer surplus earned by the most optimistic type \( t \) consumer when she purchases at time \( t' < t \) is negative, or

\[
\hat{u}(t'; t) = \frac{\pi(t)t}{t - (1 - \pi(t))t'} V(t) - V(t') \leq 0
\]

for all \( t', t' < t \).

Notice that this condition reduces to \( \pi(t)V(t) \leq V(t') \) when \( t' = 0 \), which holds, as before, when \( \pi(t) \) is sufficiently small. Notice also that (3) clearly holds with equality when \( t' = t \). So it follows that (3) holds, and condition (iii) in Proposition 4 holds, if the deviation payoff for the most optimistic consumer, \( \hat{u}(t'; t) \), is increasing in \( t' \), or

\[
\frac{d\hat{u}(t'; t)}{dt'} = \frac{\pi(t)(1 - \pi(t))t}{(t - (1 - \pi(t))t')^2} V(t) - V'(t') > 0
\]

for all \( t' \in [0, t] \). Note that

\[
\frac{d^2\hat{u}(t'; t)}{dt'^2} = \frac{2\pi(t)(1 - \pi(t))t^2 t'}{(t - (1 - \pi(t))t')^3} - \frac{V''(t')}{V(t)}
\]

is clearly positive, if \( V''(t') \leq 0 \). That is, if \( V''(t') \leq 0 \), then \( u(t'; t) \) is convex for all \( t' \in [0, t] \). Clearly \( \hat{u}(0; t) \leq 0 \) (because \( \pi(t) V(t) \leq V(0) \)) and \( \hat{u}(t; t) = 0 \) (because \( p(t) = V(t) \)), so \( \frac{d^2\hat{u}(t'; t)}{dt'^2} > 0 \) for all \( t' \in [0, t] \) implies that \( \hat{u}(t'; t) \leq 0 \) for all \( t' \in [0, t] \) and that the incentive compatibility constraint holds. So \( V''(t') \leq 0 \) implies that (3) holds, and that condition (iii) in Proposition 4 holds.\(^4\)

The example assumes information is only bad news, not good news. Obviously a good news model would not satisfy Condition (iii) of Proposition 4. The bad-news model nevertheless highlights that it is not the instantaneous learning assumption that gives the firm pricing power.

6. Conclusion

This paper considers a monopolist selling to heterogeneous, privately-informed consumers and shows that if consumers learn their valuations at different times and consumers who learn later have higher valuations conditional on wanting the good, then it may be possible to perfectly price discriminate with a sequence of increasing spot prices. The results relied strongly on the assumption that valuations have a binary distribution (in contrast with [1]).

One of the main advantages of the binary distribution assumption, is the ability to make more general learning assumptions. We showed that the basic results are robust to relaxations of the instantaneous learning assumption made in [1]. Formally, we derive a cap on the beliefs of the most optimistic consumer that assures the first-best can still be achieved when consumers learn gradually. Our assumption is easily satisfied in a bad-news example.

\(^4\)Note that time is merely an ordering, so some readers might think that we could make \( V(t) \) concave simply by rescaling the time variable. However, it is not possible to do this without changing the information structure.
Appendix A. Proof of Lemma 1.

First, we show that given an optimal mechanism, we can construct a new optimal mechanism in which payments are only made by consumers who get the good. Then we will show that this new mechanism can be implemented as a sequence of spot prices.

Let the allocation rule $y(t, v) \in \{0, 1\}$ and the payment schedule $x(t, v) \in \mathbb{R}^+$ be an optimal mechanism, where we assume (for simplicity) that $y$ is deterministic.

Because the mechanism $x, y$ is optimal, it must be incentive compatible, which implies $y(t, v)$ must be weakly increasing in $v$.

Because the mechanism $x, y$ is optimal, it also follows that $y(t, 0) = 0$, $\forall t$. Suppose not. Then consider the alternative mechanism, $\hat{x}, \hat{y}$, where $\hat{x}(t, v) = x(t, v), \forall t, v$ and $\hat{y}(t, V(t)) = y(t, V(t)), \forall t$, and $\hat{y}(t, 0) = 0, \forall t$. When she reports truthfully, the consumer surplus is unchanged because the payments are the same and the allocation is the same when her valuation is non-zero. The only difference is that under the alternative mechanism she gets the good less often when her valuation is 0. Getting the good less often can only lower her surplus when she misreports her type, so this alternative mechanism is incentive compatible and individually rational. Finally, the firm’s revenue is unchanged, but its production costs are lower, so its profits are higher, which is a contradiction.

If $y(t, V(t)) = 0$ for any $t$, then a consumer that reports $t$ never gets the good, so individual rationality implies $x(t, v) = 0$ for all $v$. Individual rationality also implies that if $y(t, V(t)) = 1$ for any $t$, then $E[y(t, v)] = (1 - \pi(t)) x(t, 0) + \pi(t) x(t, V(t)) \leq \pi(t) V(t)$ (where all expectations are taken over $v$ holding $t$ fixed).

Now consider the alternative mechanism, $\hat{x}, \hat{y}$, where $\hat{y}(t, v) = y(t, v), \forall t, v$, and if $y(t, V(t)) = 1$ then $\hat{x}(t, 0) = 0$ and $\hat{x}(t, V(t)) = ((1 - \pi(t))/\pi(t)) x(t, 0) + x(t, V(t))$, and if $y(t, V(t)) = 0$ then $\hat{x}(t, 0) = 0$ and $\hat{x}(t, V(t)) = 0$. By construction the payment $\hat{x}(t, 0) = 0$ if the consumer reports a zero valuation, so the alternative mechanism, $\hat{x}, \hat{y}$, specifies payments only for consumers who get the good.

Note that $E[y(t, v)] = E[y(t, v)]$, so $E[x(t, v)] \leq \pi(t) V(t)$ implies $E[\hat{x}(t, v)] \leq \pi(t) V(t)$, so individual rationality still holds. And the expected revenue is the same and the allocation is the same, so the expected profits are the same.

The alternative mechanism, $\hat{x}, \hat{y}$, is also incentive compatible. First, it is clear that conditional on reporting her type $t$ truthfully, the consumer reports $v$ truthfully at time $t$ since $E[y(t, v)] \leq \pi(t) V(t)$ and so $\hat{x}(t, 0) = 0$ implies $\hat{x}(t, V(t)) \leq V(t)$.

Second, consider a deviation in which a type $t$ reports $t' > t$. After such a deviation, the consumer can report her valuation as 0 or $V(t')$, and can do so conditional on her realized valuation which is 0 or $V(t)$. So there are four incentive compatibility constraints which hold for the mechanism $x, y$. The consumer who deviates to $t'$ must not be able to increase her surplus by reporting a 0 valuation regardless of her valuation (but this deviation yields 0 surplus); must not be able to increase her surplus by reporting a high valuation when it is 0 and reporting 0 when it is high (but this deviation also yields 0 surplus); must not be able to increase her surplus by reporting 0 when the true valuation is 0 and $V(t')$ when the true valuation is $V(t)$, or

$$\pi(t) V(t) y(t, V(t)) - E[x(t, v)] \geq \pi(t) V(t) y(t', V(t')) - E[x(t', v)]; \quad \text{(A.1)}$$

and must not be able to increase her surplus by reporting $V(t')$ regardless of her valuation, or

$$\pi(t) V(t) y(t, V(t)) - E[x(t, v)] \geq \pi(t) V(t) y(t', V(t')) - x(t', V(t')). \quad \text{(A.2)}$$

Clearly (A.1) implies (A.2).

Note also that $\hat{x}(t, 0) = 0$ and $\hat{x}(t, V(t)) = ((1 - \pi(t))/\pi(t)) x(t, 0) + x(t, V(t))$, so it follows that $E[x(t, v)] = (1 - \pi(t)) x(t, 0) + \pi(t) x(t, V(t)) = (1 - \pi(t)) \hat{x}(t, 0) + \pi(t) \hat{x}(t, V(t)) = E[y(t, v)]$. We now show that deviating to $t'$ does not increase the consumers’ surplus under the mechanism $\hat{x}, \hat{y}$. The first two constraints are clearly satisfied because these deviations again yield 0 surplus. The third constraint is

$$\pi(t) V(t) y(t, V(t)) - E[y(t, v)] \geq \pi(t) V(t) y(t', V(t')) - E[y(t', v)] \quad \text{(A.3)}$$

and the fourth constraint is

$$\pi(t) V(t) y(t, V(t)) - E[y(t, v)] \geq \pi(t) V(t) y(t', V(t')) - \hat{x}(t', V(t')). \quad \text{(A.4)}$$
And as above, (A.3) implies (A.4). More importantly, (A.1) implies that

\[
\pi(t)V(t)y(t, V(t)) - \mathbb{E}_v[x(t, v)] \geq \pi(t) \left[ V(t)y(t', V(t')) - \frac{(1 - \pi(t))}{\pi(t)} x(t', 0) - x(t', V(t')) \right]
\]

or

\[
\pi(t)V(t)y(t, V(t)) - \mathbb{E}_v[x(t, v)] \geq \pi(t) \left[ V(t)y(t', V(t')) + \left( \frac{1 - \pi(t)}{\pi(t')} - \frac{1 - \pi(t)}{\pi(t)} \right) x(t', 0) - \hat{x}(t', V(t')) \right]
\]

and since \( y(v, t) = \hat{y}(v, t) \) and \( \mathbb{E}_v[x(t, v)] = \mathbb{E}_v[\hat{x}(t, v)] \), it follows that

\[
\pi(t)V(t)\hat{y}(t, V(t)) - \mathbb{E}_v[\hat{x}(t, v)] \geq \pi(t)V(t)\hat{y}(t', V(t')) - \mathbb{E}_v[\hat{x}(t', v)] + \left[ \frac{1 - \pi(t)}{\pi(t')} - \frac{1 - \pi(t)}{\pi(t)} \right] x(t', 0)
\]

so (A.3) holds if the bracketed term is positive, or \( \pi(t') < \pi(t) \), or \( \pi(t) \) is a decreasing function.

Finally, consider a deviation in which type \( t \) reports type \( t'' < t \). Incentive compatibility of \( y, x \) implies two conditions, not four, because for these deviations the consumer must choose what \( v \) to report without conditioning on her realized valuation. First the consumer must not be able to increase her surplus by reporting that her valuation is 0. But this is obvious because this yields a negative surplus (the consumer never gets the good and may be required to pay). Second the consumer must not be able to increase her surplus by reporting that her valuation is \( V(t'') \), or

\[
\pi(t)y(t, V(t)) - \mathbb{E}_v[x(t, v)] \geq \pi(t)V(t)y(t'', V(t'')) - x(t'', V(t''))
\]

which clearly implies

\[
\pi(t)\hat{y}(t, V(t)) - \mathbb{E}_v[\hat{x}(t, v)] \geq \pi(t)V(t)\hat{y}(t'', V(t'')) - \hat{x}(t'', V(t''))
\]

because \( y(v, t) = \hat{y}(v, t) \), \( \mathbb{E}_v[x(t, v)] = \mathbb{E}_v[\hat{x}(t, v)] \), and \( \hat{x}(t'', V(t'')) \geq x(t'', V(t'')) \) by construction. So \( \hat{y}, \hat{x} \) is incentive compatible.

We now argue that \( \hat{y}, \hat{x} \) can be implemented as a sequence of spot prices \( p(t) = \hat{x}(t, V(t)) \). A consumer who prefers reporting \( t \) to reporting \( t' < t \) at time 0 will also prefer to report \( t \) at time \( t' \) since she will have not learned any new information at time \( t' \) that will effect her decision. And a consumer who prefers time reporting time \( t \) to reporting time \( t' > t \) will either learn her valuation is 0 or that it is \( V(t) \). If she learns her valuation is 0, she will not report it is \( V(t) \) at time \( t \) or report that it is \( V(t') \) at any later time \( t' \), so the sequential prices are incentive compatible. If she learns her valuation is \( V(t) \), then (A.3) implies

\[
\pi(t)V(t)\hat{y}(t, V(t)) - \pi(t)\hat{x}(t, V(t)) \geq \pi(t)V(t)\hat{y}(t', V(t')) - \pi(t)\hat{x}(t', V(t'))
\]

so

\[
V(t)\hat{y}(t, V(t)) - \hat{x}(t, V(t)) \geq V(t)\hat{y}(t', V(t')) - \hat{x}(t', V(t'))
\]

or, since we have assumed \( x(t, V(t)) = \hat{x}(t, V(t)) = 1 \) for all \( t \),

\[
p(t) = \hat{x}(t, V(t)) \leq \hat{x}(t', V(t')) = p(t')
\]

so the consumer purchases at time \( t \). (But note that \( \hat{x}(t, V(t)) = 0 \) if the consumer does not get the good when she reports truthfully, that is, if \( \hat{y}(t, V(t)) = 0 \), and in this case (A.3) implies \( p(t') > V(t) \) for all \( t' > t \), so the prices are still incentive compatible.)
References


