Bundling Can Signal High Quality*

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June 8, 2018

Abstract

Bundling experience goods can signal high quality in a simple static model. Bundling is a useful signal of high quality when consumers are only partially informed about quality, because it makes imitation by a firm with one or two low-quality products more costly. As with high prices, bundling can also signal high quality by restricting sales, because restricting sales less costly for a high-cost, high-quality firm, but signaling with high prices is always a more profitable way to restrict sales.

*I would like to thank Yongmin Chen, Frances Lee and especially Kathryn Spier for helpful comments.

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1 Introduction

Product bundling can signal that a firm’s experience goods are high quality, even in simple static settings. This paper contributes to an important literature on the potential efficiency benefits of product bundling and formalizes an argument that is commonly used by defendants in antitrust cases.

A good example of a product quality defense in a legal antitrust case is the Hilti corporation. Hilti is a leading producer of nail guns and supplies, and it defended its bundling practices arguing that the using a competitor’s nails would “give rise to uncertain fixing reliability and, consequently, safety risks in load bearing applications.” And there are many other examples in the automobile and computer industries.

I show that bundling can act as a signal by making it more costly for low-quality firms to imitate high-quality firms. Bundling is costly for firms with low-quality products, because when consumers have noisy information about product quality, then one negative signal about product quality will deter the consumer from purchasing the bundle, but, absent bundling, the consumer might otherwise have purchased the other product.

Bundling may also restrict output, so, like high prices, bundling can also signal that the firm is a high-quality firm even when consumers receive no information about product quality. However, I show that raising price is always a more effective way to signal high quality than other methods of restricting output, such as bundling. More specifically, other methods of restricting output do not

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satisfy the intuitive criterion.

In my main analysis, I show that when consumers are partially informed about product quality, then consumers who know one product is low quality won’t buy either product when they are bundled. This implies that bundling can be costless for a firm with high-quality products, but costly for firms with one or two low-quality products. So bundling is a useful signal of product quality.

The paper is organized as follows. Section 2 discusses some of the related literature. Section 3 describes the model. Section 4 considers the one-product firm and also shows that restricting output can signal high quality, but doing so with price always dominates. Section 5 considers the two-product firm and product bundling. Finally, Section 6 provides some concluding remarks and discussion.

2 Literature on Bundling

Many research papers have focused on the reasons for product bundling, but few papers have considered bundling’s impact on product quality. Much of the work on bundling emphasizes price discrimination. Adams and Yellen (1976), McAfee, McMillan and Whinston (1989), and Fang and Norman (2006) show bundling can help firms to extract greater surplus from consumers. Elhauge and Nalebuff (2014) show that bundling or tying allows firms to use consumption of a tied nondurable goods as a proxy for consumers’ valuation of a durable good.

Another important strand of the bundling literature considers the use of bundling to extend market power. Whinston (1990), Nalebuff (2004), and others show that bundling can be used to foreclose competition (see the survey by

A few papers suggest that bundling can increase quality by solving an attribution externality. In the law and economics literature, Bork (1978) and Posner and Easterbrook (1981) suggest that when consumers use low-quality products, then low overall performance may be erroneously attributed to the producer of related high-quality products, and so tying or bundling products together can eliminate this externality. But as they point out, consumers don’t necessarily need to have their choices constrained to solve this problem – they typically have private incentives to sole source when purchasing. Iacobucci (2003) more formally highlights the importance attribution problem (see also Bar-Gill, 2006).

Dana and Spier (2017) show that product bundling can lead to higher quality by improving monitoring even in the absence of the attribution problem. Bundling can improve both the quality of monitoring and the effectiveness of consumer punishment strategies. The show that bundling is even more effective in the presence of an attribution externality. Dana and Spier (2015) make the related point that minimum purchase requirements can improve monitoring in a simple homogeneous-good model.

Two other papers consider models in which product bundling signals high-quality products, but both of these papers exploit dynamic aspects of the consumers’ purchase decisions. These papers assume the consumer can stop purchasing the one good (e.g., a non-durable) after observing the quality of the other (e.g., a durable). First, Schwartz and Werden (1996) consider a signaling model
in which a privately-informed firm uses tying to signal the quality of its durable
good. By tying the sale of the durable good to a non-durable good, the firm can
shift the rents from the durable to the non-durable and overcome the hidden-
information problem. Second, Choi (2003) shows that a firm with an established
branded product can bundle it with a new product to signal quality. By irre-
versibly committing to bundle its products, the firm is using its future rents on
the established product as a bond to signal quality of the new product – bundling
constrains consumers to stop purchasing both products when they learn that the
new product is low quality.

Several papers are related that consider the use of prices as a signal of high
quality, most notably Bagwell and Riordan (1991), though the point was first
made by Wilson (1980). That paper mainly emphasizes that a high price signals
high quality because it restricts output. While it also allows consumers to have
noisy signals of quality, that fact does not make price a signal of quality. This
paper extends Bagwell and Riordan and argues that because it restricts output
and constrains consumers to essential purchase neither good when they observe
one low quality good, bundling can also be useful as a signal of quality.

In addition, Ellingsen (1997) shows that price can signal high quality even
when demand is perfectly inelastic. When price is equal to consumers’ valuation,
then consumer mixing (some buy, other don’t) can restrict sales and hence signal
high quality. In an earlier version of my paper, I showed that bundling can restrict
sales with inelastic demand by selecting only consumers who want both goods,
and so signal high quality in a related way. And Linnemer (2002) extends Bagwell
and Riordan (1991) to allow both price and advertising as signals of quality.
3 The Model

A single firm sells two products, product 1 and product 2. With probability $\theta$ product 1 is a high-quality, and with probability $1 - \theta$ product 1 is a low-quality product. The same is true for product 2. High-quality products cost $c_h$ per unit and low-quality products cost $c_l$ per unit where $c_h > c_l$. The quality of the firm’s two products is independently distributed, which implies that the firm has four types: $ll$, $lh$, $hl$ or $hh$.

Consumers may have noisy signals of quality. Before consumers make their purchase decisions, a fraction $x$ of all consumer learn the product quality of each good. I assume that this probability is independently distributed, so a fraction $x^2$ of consumers know the product quality of both goods, that a fraction $2x(1 - x)$ know the product quality of exactly one good, and that a fraction $(1 - x)^2$ know the product quality of neither good.

The firm chooses whether or not to bundle its products and what price to charge for each product, or what price to charge for the bundle. All consumers have a common willingness to pay $v_l > c_l$ for a low-quality product.

Consumers’ willingness to pay for a high-quality product is equal to $v_h = v_l + t$ where $t$ is uniformly distributed the $[0, 1]$ interval. So $v_h$ is uniformly distributed on $[v_l, v_l + 1]$, which I also write as $[\underline{v}_h, \bar{v}_h]$.

While I assume that a consumers’ willingness to pay is heterogeneous across consumers, I assume it is homogeneous across goods, so a consumer who is will-

\footnote{While Bagwell and Riordan (1991) considered just one product, I use the same distribution of valuations as they did in their model. The other modeling assumptions are also the same, except that Bagwell and Riordan normalized the cost parameter $c_l$ to 0. Bagwell and Riordan also used $X$ to denote the ratio of informed consumers to uninformed consumers, which corresponds to $x/(1 - x)$ in my notation.}
ing to pay \( v_h \) for a high-quality version of product 1 is willing to pay \( v_h \) for a high-quality version of product 2 and is willing to pay \( 2v_h \) for a bundle of two high-quality goods. This assumption simplifies the firm’s pricing decision. Also, I have implicitly assumed that the two products are neither complements nor substitutes, so consumers’ willingness to pay is independent of whether or not they consume the other good.

I assume that both products create surplus (trade is efficient), or equivalently that \( v_l > c_l \) and \( v_l + 1 > c_h \), and also that \( v_l + 1 - c_h > v_l - c_l \), so the high-quality product creates more surplus than the low-quality product for some consumers. I also assume that \( 1 + c_h > v_l \), which, with the other assumptions, implies that \( c_h \in [v_l - 1, v_l + 1] \), and, more importantly, is equivalent to assuming that under complete information the high-quality good is sold at a strictly higher price than the low-quality good.

Let \( \pi_i(p, \psi) \) denote the firm’s profit function for each product if it does not bundle its products, and let \( \pi_i^b(p, \psi) \) denote the firm’s profit function if it bundles its products, where \( i \in \{l, h\} \) and \( i \in \{ll, lh, hl, hh\} \) denote the firm’s type, where the price \( p \) and the consumers’ beliefs \( \psi \) denote a scalar in the first case and a vector of the firm’s prices in the second case. For simplicity, I will abuse this notation and write the vectors as scalars when the prices and beliefs are the same for both goods.

Finally, I assume \( v_l + c_l \leq 1 \). This implies that there exists some price \( p \) greater than \( v_l \) such that \( \pi_i(p, 1) > \pi_i(v_l, 0) \). Otherwise the low-quality firm never has an incentive to imitate the high-quality firm, and the complete-information prices are always an equilibrium.
4 The Single-Product Firm Benchmark

The model and results in this section closely follow Bagwell and Riordan (1991). Let \( \pi_i(p, \psi) \) denote single-product firm’s profit function, where \( i \in \{l, h\} \) is the firm’s type, \( p \) is the price, and \( \psi \) is consumers’ beliefs about the probability that the firm is the high-quality firm.

The complete-information prices are \( p_l^* = v_l \) and \( p_h^* = (1 + v_l + c_h)/2 \). Under incomplete information, a separating equilibrium exists in which the high-quality firm charges \( p_h^* \) and the low-quality firm charges \( p_l^* \) as long as the low-quality firm prefers to charge \( p_l^* \), which is true if \( \pi_l(p_l^*, 0) \geq \pi_l(p_h^*, 1) \), and the high-quality firm prefers to charge \( p_h^* \), which is true if \( \pi_h(p_h^*, 1) \geq \pi_h(p_l^*, 0) \). This equilibrium is supported by beliefs that the firm is low-quality if it sets any off-the-equilibrium path price.

Clearly the second condition holds since \( p_h^* \) is the high-quality firm’s complete information profit maximizing price. The first condition can be written as

\[
p_l^* - c_l \geq (1 - x) \left( p_h^* - c_l \right) (\bar{v}_h - p_h^*), \tag{1}
\]

or

\[
v_l - c_l \geq (1 - x) \left( \frac{1 + v_l + c_h}{2} - c_l \right) \left( \frac{1 + v_l - c_h}{2} \right). \tag{2}
\]

And the beliefs satisfy the intuitive criterion because both types are earning the highest possible, complete-information profits.

This implies the following result:
**Proposition 1.** A perfect Bayesian equilibrium of the game satisfying the intuitive criterion exists in which the firm charge the complete-information prices if and only if equation (2) holds.

The following corollary is obvious from (2).

**Corollary 1.** There exists a cutoff $x_1 \geq 0$ such that a perfect Bayesian equilibrium of the game satisfying the intuitive criterion in which the firm charge the complete-information prices exists if and only if $x \geq x_1$.

Less obviously, a perfect Bayesian equilibrium of the game satisfying the intuitive criterion in which the firm charges the complete-information prices may exist even if $x = 0$, as long as $c_h$ is sufficiently larger than $c_l$. That is, when the low-cost firm’s cost advantage is large, then the low-cost firm may prefer to sell to all of the consumers at a smaller profit margin rather than to sell to a very small number consumers at a much larger profit margin. For example, suppose that $x = 0$, $c_l = .6$, $v_l = 1$, and $c_h = 1.5$. Then the complete-information prices are 1 and 1.75 and the profits at 1 are .4 while the profits at 1.75 are lower, and equal to $.25 \times 1.15 = .2875$, even when $x = 0$. The intuition that setting a high price restricts output and that this signals quality when $c_h > c_l$ is originally due to Wilson (1980x) and is emphasized in Bagwell and Riordan (1991).

So high prices can signal high quality either because the high-quality firm’s costs or higher than the low-quality firm’s costs, or because $x > 0$. That later does not restrict output, but does imply that the low-quality firm looses some of its consumers when it imitates the high-quality firm.
Proposition 2. When the complete-information prices can be supported as a perfect Bayesian equilibrium satisfying the intuitive criterion, then these prices are the unique equilibrium prices.

Proof. Clearly the equilibrium is the unique separating equilibrium satisfying the intuitive criteria. The low quality firm prefers to charge its complete-information price regardless of what consumers believe, and, given this, no price for the high-quality firm other than the complete-information price can satisfy the intuitive criterion.

In addition, no pooling equilibrium satisfying the intuitive criterion exists when (2) holds. This is because the low-quality firm’s profits in any pooling equilibrium must be higher than its profits in the separating equilibrium, otherwise it would deviate to \( p_l^* \), so by (2), the low-quality firm’s profits are higher than \( \pi_l(p_h^*, 1) \), and so by the intuitive criterion, so any pooling equilibrium does not satisfy the intuitive criterion - consumers cannot believe that a firm that charges \( p_h^* \) has a low-quality product, but if they believe a firm that charges \( p_h^* \) has a high-quality product the high-quality firm has a profitable deviation.

So firms charge the complete-information prices in every perfect Bayesian equilibrium.

Equation (2) is not always satisfied, but it is clearly satisfied when \( c_h \) is sufficiently large – the first term is positive and finite for all \( c_h \in [v_l - 1, v_l + 1] \), and the second term goes to zero as \( c_h \) goes to \( 1 + v_l \) from below, so for sufficiently large \( c_h \) the right hand side goes to zero.

When the complete-information prices are not feasible, then a separating equilibrium may still exist. Clearly necessary conditions for existence of a separating
equilibrium in which the high-quality firm charges a price $p$ and the low-quality firm charges $p_l^*$ are $\pi_l(p_l^*, 0) \geq \pi_l(p, 1)$, and $\pi_h(p, 1) \geq \pi_h(p_l^*, 0)$. If these conditions hold for some price $p$ then they can be supported as a perfect Bayesian equilibrium with the beliefs that the firm is low-quality at any other price, but only one price yields a separating equilibrium which satisfies the intuitive criterion.

Let $\hat{p}_h(x)$ denote the larger of two values implicitly defined the low-quality firm’s indifference condition, or $\pi_l(p_l^*, 0) = \pi_l(\hat{p}_h(x), 1)$, or equivalently, by

$$v_l - c_l = (1 - x) \left( \hat{p}_h(x) - c_l \right) (1 + v_l - \hat{p}_h(x)).$$  \hspace{1cm} (3)

Clearly (3) has two solutions. Both solutions are positive and less than $1 + v_l = \bar{v}_h$. The larger of the two prices is more profitable for the high-quality firm because, as is easy to check, the larger price is closer to the complete-information price.

This implies that a separating equilibrium in which the high-quality firm charges the smaller of the two solutions (and any price below it) cannot satisfy the intuitive criterion – the high quality firm prefers to offer $\hat{p}_h(x)$ and the belief that a firm offering $\hat{p}_h(x)$ does satisfy the intuitive criterion because the low-quality firm’s profits would be lower even if consumers believed it were high quality. Similarly prices between $\hat{p}_h(x)$ and $\bar{v}_h$ are not be possible in a separating equilibrium satisfying the intuitive criterion. And prices between the two solutions are not possible in a separating equilibrium because the low-quality firm would imitate the high-quality firm.

The following result (Bagwell and Riordan, Theorem 1) follows immediately.
Proposition 3. For the single-product benchmark model, if a separating equilibrium satisfying the intuitive criterion exists, the low-quality firm’s price is \( p_l^* = v_l \) and the high-quality firm’s price is \( p_h = \max \{ p_h^*, \hat{p}_h(x) \} \).

The separating equilibrium described in Proposition 3 exists if \( \pi_h(\hat{p}_h, 1) \geq \pi_l(p_l^*, 0) \), or the high-quality firm prefers to charge \( \hat{p}_h \) as opposed to \( p_l^* \). Sufficient conditions for existence are described in detail in Bagwell and Riordan (1991). Of course, pooling equilibria may also exist.

Bagwell and Riordan (1991) analyzed a firm that used price as a signal of quality, not other decisions such as bundling.\(^3\) Before turning to the two-product firm, I first explore whether alternative ways for the single-product firm to signal high quality are profitable.

High price works as a signal because it restricts output, so in particular other actions by the firm that restricts its sales may also be able signal a high-quality product. In fact, it is easy to show that this is the case. But rather than demonstrate what is feasible, I will instead prove a negative result. I now show that other signals that restrict output are inferior to signaling with a high price in the sense that they don’t satisfy the intuitive criterion. Put another way, signaling with a high price is always more profitable.

Suppose that the complete-information prices are not feasible, and that a separating equilibrium exists in which the firm sets prices \( \hat{p}_h(x) \) and \( p_l^* \). Can the

\(^3\) Bundling can easily restrict sales. Consider a firm facing independently distributed uniform demand on the interval \([1, 2]\) for each of its products, with cost equal to 1.5 for both of its products. The firm would charge 1.75 without bundling and it sell each good to 1/4 of the consumers. If the firm bundled its products then its sales would be strictly lower. Sales are lower at any price above cost, so in equilibrium sales are lower. If the firm bundled and charged 3.5 for the bundle, it would sell the bundle of two goods to 1/8th of the consumers. Total sales fall from 1/2 to 1/4.
firm do better by restricting its sales in other ways besides setting a high price?

To answer this question, I now look for separating equilibrium in a new enlarged game in which the firm chooses both a price and an upper bound on its sales.

**Proposition 4.** *In any separating equilibrium satisfying the intuitive criterion of the enlarged game in which the firm can restrict or ration its sales, the firm signals high quality by charging $\hat{p}_h(x)$ and does not restrict its sales.*

The proof of Proposition 4 and the proofs of other remaining propositions are in the appendix.

Proposition 4 is important because it reveals that if bundling is going to be used as a signal of high quality, it is not simply because it restricts sales. Signaling using just a high price is a more profitable strategy for the firm.

## 5 The Two-Product Firm and Bundling

In this section I show that bundling is an effective way to signal high quality when consumers are imperfectly informed about quality. That is, bundling signals high quality when $x > 0$, because bundling makes it more difficult for the low-quality firm to imitate the high-quality firm. This differs from signals (like price) that work because they restrict output. In this model bundling has no impact on the high quality firms output, because, by assumption, consumers’ have the same valuation for each product.

Notice that the single-product firm analysis in the previous section also describes the pricing decisions of the two-product firm. That is, if a two-product
firm cannot bundle its products, then it would have face the same decision problem and make the same choices as two one-product firms. But a two-product firm that can bundle will do so, because bundling can increase its profits.

The easiest way to show that bundling is useful is by showing that bundling can make the complete-information prices feasible for the high-quality firm when the complete-information prices are not feasible without bundling.

Consider a separating equilibrium in which the \( hh \) firm bundles its products. The complete-information prices can be supported as a separating equilibrium satisfying the intuitive criterion if two condition hold. First, none of the firms with low-quality products want to imitate the \( hh \) firm, which is true if \( \pi^b_{lh}(p^*_h, 0) \geq \pi^b_{lh}(p^*_l, 1) \); \( \pi^b_{hl}(p^*_l, 0) \geq \pi^b_{hl}(p^*_h, 1) \); and \( \pi^b_{ll}(p^*_l, 0) \geq \pi^b_{ll}(p^*_h, 1) \). And second, the \( hh \) firm does not want to imitate the low-quality firm, or \( \pi^b_{hh}(p^*_h, 0) \geq \pi^b_{hh}(p^*_l, 0) \). (In addition 1 must hold – the \( ll \) firm does not want to imitate the \( lh \) or \( hl \) firms.)

No other deviation is profitable when consumers have the most pessimistic off-the-equilibrium-path beliefs, and clearly they satisfy the intuitive criterion.

**Proposition 5.** There exists a value \( x_2 \leq x_1 \) such that for all \( x \geq x_2 \), a separating equilibrium satisfying the intuitive criterion exists in which all firms charge the complete-information prices, and so bundling leads to lower prices and greater profits for \( x \in [x_2, x_1] \), where \( x_2 \) is strictly less than \( x_1 \) when \( x_1 > 0 \).

Proposition 5 establishes that bundling makes it easier to sustain the complete-information prices. For all \( x \in [x_2, x_1) \) the complete-information prices can be sustained only if the \( hh \) firm bundles. In this range, bundling lowers prices, increases profits, and makes everyone better off.

Now suppose that even with bundling the complete-information prices are
not feasible. That is, suppose \( x < x_2 \). The firm may still be able to signal high quality using high prices. I show that if a separating equilibrium exists in which the firm uses high prices to signal high quality, then a more profitable separating equilibrium also exists in which the firm signals high quality with a combination of high prices and bundling, and moreover only the bundling separating equilibrium satisfies the intuitive criterion.

**Proposition 6.** Suppose that the complete-information prices are not feasible, but that a separating equilibrium exists in which the firm charges \( \hat{p}_h(x) > p_h^* \) for its high quality products and \( p_l^* \) for its low quality products. Then a higher profit equilibrium satisfying the intuitive criterion also exists in which the hh firm bundles and charges some price \( \hat{p}_{hh}^b(x) \) strictly less than \( \hat{p}_h(x) \).

It is also easy to show that \( \hat{p}_{hh}^b(x) \) is uniquely defined, and that, as before, bundling makes consumers and the firm strictly better off.

6 Conclusion

The paper shows that bundling can signal high quality in a simple static model. While other papers make a similar point in more dynamic models that might even fit applications better, this paper shows the value of bundling in a simpler setting. It also highlights the two main mechanisms that bundling uses. First, bundling can restrict sales which is more attractive for high-cost/high-quality firms than for low-cost/low-quality firms. And second, bundling makes it more expensive to consume just one product, so when consumers have noisy information, bundling can raise the cost to the low-quality firm of imitating the high-quality firm.
References


Appendix

Proof of Proposition 4

Proof. Suppose that a separating equilibrium satisfying the intuitive criterion exists in which the high-quality firm charges some price \( \tilde{p}_h \) per unit sold of each good and restricts its sales to some quantity \( \tilde{q}_h \) units of the good. I now show that \( \tilde{q}_h \geq \bar{v}_h - \tilde{p}_h \), so the high-quality firm signals high quality with a high price and not by restricting output.

Suppose not. That is, suppose that \( \tilde{q}_h < \bar{v}_h - \tilde{p}_h \). Consider another price and quantity, \( p \) and \( q \), defined by the intersection of the low-quality firm’s equal-profit condition, \( (1 - x)(p - c_l)q = (1 - x)(\tilde{p} - c_l)\tilde{q} \), and by high-quality firm’s demand, \( q = \bar{v}_h - p \). This intersection is clearly unique. By construction, the low-quality firm’s profits at \( p \) and \( q \) are the same as its profits at the high-quality firm’s equilibrium price and quantity. And by construction \( q \) is equal to demand, so \( q \) does not restrict the high-quality firm’s sales.

More importantly, the high-quality firm’s profits are strictly higher at price \( p \). This is because \( c_h > c_l \), and \( q < \tilde{q}_h \), so \( (p - c_l)q = (\tilde{p} - c_l)\tilde{q} \) implies \((p - c_l)q - (c_h - c_l)q = (\tilde{p} - c_l)\tilde{q} - (c_h - c_l)q\), which implies \((p - c_h)q = (\tilde{p} - c_h)\tilde{q} + c_h(\tilde{q}_h - q)\), and so \((p - c_h)q > (\tilde{p} - c_h)\tilde{q}\). That is, the high-quality firm earns higher profits at \( p \) and \( q \).

The deviation is profitable for the high-quality firm and is not profitable for the low-quality firm, even if consumers believe the firm is the high-quality firm. So the proposed equilibrium does not satisfy the intuitive criterion, which is a contradiction. Therefore \( \tilde{q}_h \geq \bar{v}_h - \tilde{p}_h \) for any separating equilibrium satisfying
the intuitive criterion. 

**Proof of Proposition 5**

*Proof.* To prove the result we only need to check that the two conditions described above hold.

Demand for the bundle is easy to characterize. If a consumer believes both products are high quality, then the demand for the bundle is $1 + v_l - p^*_h$. If a consumer observes that one product is low quality and believes the other product is high quality then he or she will purchase the bundle if $v_l + v_l + t \geq 2p^*_h$, or $t \geq 2p^*_h - 2v_l$, so the demand for the bundle is $1 - 2p^*_h + 2v_l$. If a consumer observes that both products are low-quality, he or she will never purchase, since $p^*_h > v_l$, so demand for the bundle is zero.

If the $hh$ firm does not bundle its products, and the $ll$ firm imitates the $hh$ firm, then $(1 - x)(1 + v_l - p^*_h)$ consumers purchase each product at a price $p^*_h$ and the rest don’t purchase, either because the price is too high or because they think the product is low quality, so imitation is not profitable if

$$2(v_l - c_l) \geq 2(1 - x)(p^*_h - c_l)(1 + v_l - p^*_h),$$

(A1)

which is identical to equation (1).

If the $hh$ firm does not bundle its products, and the $lh$ firm (or $hl$ firm) imitates the $hh$ firm, then $(1 - x)(1 + v_l - p^*_h)$ consumers purchase the low-quality good for $p^*_h$ and the rest don’t purchase (because the price is too high or they think
the product is low quality), so imitation is not profitable if

$$v_l - c_l + (p^*_h - c_h)(1 + v_l - p^*_h) \geq (1 - x)(p^*_h - c_l)(1 + v_l - p^*_h) + (p^*_h - c_h)(1 + v_l - p^*_h), \quad (A2)$$

which is also identical to equation (1).

If the $hh$ firm does bundle its products, and the $ll$ firm imitates the $hh$ firm, then $(1 - x)^2(1 + v_l - p^*_h)$ consumers think both products are high quality and pay $2p^*_h$ for the bundle; $x(1 - x)(1 + v_l - 2p^*_h)$ consumers observe that one of the products is low quality and think the other product is high quality, and they pay $2p^*_h$ for the bundle; and the rest of the consumers do not purchase, either because the price is too high or because they observe that both products are low quality. So imitation is not profitable if

$$2(v_l - c_l) \geq 2(1 - x)^2(p^*_h - c_l)(1 + v_l - p^*_h) + x(1 - x)(p^*_h - c_l)(1 + 2v_l - 2p^*_h). \quad (A3)$$

The right hand side of equation (A3) can be rewritten as

$$(1 - x)((1 - x)(p^*_h - c_l)(1 + v_l - p^*_h) + x(p^*_h - c_l)(1 + 2v_l - 2p^*_h)) \quad (A4)$$

so equation (A1), or equivalently, equation (1), implies equation (A3) if

$$(p^*_h - c_l)(1 + v_l - p^*_h) \geq (1 - x)(p^*_h - c_l)(1 + v_l - p^*_h) + x(p^*_h - c_l)(1 + 2v_l - 2p^*_h), \quad (A5)$$

and this is always true because $1 + v_l - p^*_h > 1 + 2v_l - 2p^*_h$ or $p^*_h > v_l - 1$, which is true because $c_h \geq v_l - 1$.

If the $hh$ firm bundles its products, and the $lh$ firm (or $hl$ firm) imitates the
hh firm, then \((1 - x)(1 + v_l - p_h^*)\) consumers believe both products are high quality and pay \(2p_h^*\); \(x(1 + v_l - 2p_h^*)\) consumers believe one product is high quality and one product is low quality and pay \(2p_h^*\); and the other consumers do not buy. So imitation is not profitable if

\[
(v_l - c_l) + (p_h^* - c_h)(1 + v_l - p_h^*) \geq (1 - x)(p_h^* - c_l + p_h^* - c_h)(1 + v_l - p_h^*) + x(p_h^* - c_l + p_h^* - c_h)(1 + 2v_l - 2p_h^*), \tag{A6}
\]

which can be rewritten as

\[
(v_l - c_l) + (p_h^* - c_h)(1 + v_l - p_h^*) \geq (1 - x)(p_h^* - c_l + p_h^* - c_h)(1 + v_l - p_h^*) + x(p_h^* - c_l + p_h^* - c_h)(1 + 2v_l - 2p_h^*). \tag{A7}
\]

The right hand side of equation (A6) can be rewritten as

\[
(1 - x)(p_h^* - c_l)(1 + v_l - p_h^*) + x(p_h^* - c_l) (1 + 2v_l - 2p_h^*) + (1 - x)(p_h^* - c_h)(1 + v_l - p_h^*) + x(p_h^* - c_h)(1 + v_l - p_h^*), \tag{A8}
\]

so as above, equation (A1) implies that equation (A6) holds. It is also easy to see that (A3) implies (A6).

So both conditions are satisfied if equation (A3) holds. And it is clear that there exists a cutoff \(x_2\) such that (A3) holds for all \(x \geq x_2\). And comparing (A3)
to (A1) clearly implies \( x_2 < x_1 \) unless \( x_1 = 0 \).

\[\square\]

**Proof of Proposition 6**

*Proof.* Following the argument in Proposition 3, if a separating equilibrium exists then \( \pi_l(p^*_l, 0) = \pi_l(\hat{p}_h(x), 1) \), and \( \pi_h(\hat{p}_h(x), 1) \geq \pi_h(p^*_l, 0) \). The former condition, equation (3), defines \( \hat{p}_h(x) \).

The \( hh \) firm can set \( \hat{p}_h(x) \) for both products, but it can do better. Consider a separating equilibrium satisfying the intuitive criterion in which only the \( hh \) firm bundles and the \( hh \) firm charges \( 2\hat{p}_h(x) \) for the bundle. I claim that this equilibrium does not satisfy the intuitive criterion. I show below that \( \pi_l(p^*_l, 0) > \pi_l(\hat{p}_h(x), 1) \), \( \pi_h((p^*_l, \hat{p}_h(x)), (0, 1)) > \pi_h(\hat{p}_h(x), 1) \), and \( \pi_{hl}((\hat{p}_h(x), p^*_l), (1, 0)) > \pi_{hl}(\hat{p}_h(x), 1) \). If these three conditions hold, then bundling has no effect on the \( hh \) firm’s profits. But bundling by the \( hh \) firm reduces the incentive of the \( ll \), \( lh \), and \( hl \) firms to imitate the \( hh \) firm. So the \( hh \) firm can earn higher profits at a price \( \hat{p}_{hh}^b(x) \) just below \( \hat{p}_h(x) \), as long as consumers believe it is the \( hh \) firm, and moreover no other beliefs satisfy the intuitive criterion because the other types’ profits are strictly higher than the profits they could at this price, even if consumers believed they were the \( hh \) type.

Clearly the separating equilibrium that satisfies the intuitive criterion is one in which the \( hh \) firm bundles and charges the lowest price, \( \hat{p}_{hh}^b(x) \), that is above \( p_h^* \) and satisfies: \( \pi_l(p^*_l, 0) \geq \pi_l(\hat{p}_{hh}^b(x), 1) \), \( \pi_{lh}((p^*_l, \hat{p}_h(x)), (0, 1)) \geq \pi_{lh}(\hat{p}_{hh}^b(x), 1) \), and \( \pi_{hl}((\hat{p}_h(x), p^*_l), (1, 0)) \geq \pi_{hl}(\hat{p}_{hh}^b(x), 1) \). By construction, the \( ll \), \( lh \), and \( hl \) firms have no incentive to imitate the \( hh \), and there is no deviation for the \( hh \) that would increase its profits if consumers believed it where the \( hh \) firm and that
would not be profitable for the \( ll \), \( lh \), or \( hl \) firm to imitate.

It is easy to check that the three conditions above all hold. First,

\[
\pi_{ll}(p^*_l, 0) - \pi_{ll}(\hat{p}_h^b(x), 1) = 2(v_l - c_l)
- 2(1 - x)^2(\hat{p}_h(x) - c_l)(1 + v_l - \hat{p}_h(x))
+ x(1 - x)(\hat{p}_h - c_l)(1 + 2v_l - 2\hat{p}_h(x)). \tag{A9}
\]

Substituting for \( v_l - c_l \) using (3), this becomes:

\[
\pi_{ll}(p^*_l, 0) - \pi_{ll}(\hat{p}_h^b(x), 1) = 2(1 - x)(\hat{p}_h(x) - c_l)(1 + v_l - \hat{p}_h(x))
- 2(1 - x)[(1 - x)(\hat{p}_h(x) - c_l)(1 + v_l - \hat{p}_h(x))
+ x(\hat{p}_h - c_l)(1/2 + v_l - \hat{p}_h(x))]
= x [(1 - x)(\hat{p}_h(x) - c_l)(1 + v_l - \hat{p}_h(x))
+ x(\hat{p}_h - c_l)(1/2 + v_l - \hat{p}_h(x))] > 0. \tag{A10}
\]

So \( \pi_{ll}(p^*_l, 0) > \pi_{ll}(\hat{p}_h^b(x), 1) \).

Similarly,

\[
\pi_{lh}((p^*_l, \hat{p}_{hh}(x)), (0, 1)) - \pi_{lh}(\hat{p}_{hh}(x), 1)
= (v_l - c_l) + (\hat{p}_h(x) - c_h)(1 + v_l - \hat{p}_h(x)) - (1 - x)(\hat{p}_h(x) - c_l + \hat{p}_h(x) - c_h)(1 + v_l - \hat{p}_h(x))
- x(\hat{p}_h(x) - c_l + \hat{p}_h(x) - c_h)(1/2 + v_l - \hat{p}_h(x)) \tag{A11}
\]
Again, substituting for $v - c_l$ using (3), this becomes

$$
\pi_{lh}((p^*_l, \hat{p}_{nh}(x)), (0, 1)) - \pi_{lh}(\hat{p}_{hh}(x), 1)
= (1 - x)(\hat{p}_{h}(x) - c_l)(1 + v_l - \hat{p}_{h}(x)) + (\hat{p}_{h} - c_h)(1 + v_l - \hat{p}_{h}(x))
- (1 - x)(\hat{p}_{h}(x) - c_l + \hat{p}_{h}(x) - c_h)(1 + v_l - \hat{p}_{h}(x))
- x (\hat{p}_{h}(x) - c_l + \hat{p}_{h}(x) - c_h) (1/2 + v_l - \hat{p}_{h}(x))
= (\hat{p}_{h} - c_h)(1 + v_l - \hat{p}_{h}(x))) - (1 - x)(\hat{p}_{h}(x) - c_h)(1 + v_l - \hat{p}_{h}(x))
- x (\hat{p}_{h}(x) - c_l + \hat{p}_{h}(x) - c_h) (1/2 + v_l - \hat{p}_{h}(x))
$$

(A12)

So

$$
\pi_{lh}((p^*_l, \hat{p}_{nh}(x)), (0, 1)) - \pi_{lh}(\hat{p}_{hh}(x), 1)
= x(\hat{p}_{h} - c_h)(1 + v_l - \hat{p}_{h}(x))) - x (\hat{p}_{h}(x) - c_l + \hat{p}_{h}(x) - c_h) (1/2 + v_l - \hat{p}_{h}(x))
= x(\hat{p}_{h} - c_h)(1/2)) - x (\hat{p}_{h}(x) - c_l) (1/2 + v_l - \hat{p}_{h}(x)) > 0
$$

(A13)

which is positive because $c_h > c_l$ and $\hat{p}_{h}(x) > v_l$. So $\pi_{lh}((p^*_l, \hat{p}_{h}(x)), (0, 1)) > \pi_{lh}(\hat{p}_{h}(x), 1)$ (and by analogy $\pi_{hl}((\hat{p}_{h}(x), p^*_l), (1, 0)) > \pi_{hl}(\hat{p}_{h}(x), 1)$ holds. \[\square\]