Using yield management to shift demand when the peak time is unknown

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Traditional peak-load and stochastic peak-load models assume firms have prior information about when peak demand occurs. However, price dispersion, such as is typically used by firms practicing yield management, can achieve some of the same efficient demand shifting even when the peak time is unknown. Equilibrium price dispersion arises because of stochastic demand and price rigidities, but a previously unexplored benefit of price dispersion is its ability to reduce equilibrium capacity costs through demand shifting. The model also suggests how yield management (now more commonly called revenue management) might actually benefit business travellers, contrary to the popular prejudice.

1. Introduction

- Firms, as well as social planners, set higher prices at peak times in order to shift demand to off-peak times and efficiently reduce capacity costs. In the peak-load pricing and stochastic peak-load pricing literature, firms exploit prior knowledge (however imperfect) about when demand peaks occur. This article shows that even if firms do not know when demand peaks will occur, they will still shift demand from peak to off-peak times by setting multiple prices and rationing availability at lower prices. Exploiting the fact that lower-priced units stock out at the peak time before they stock out at the off-peak time, a firm or planner can set the same ex ante price schedule for each period and still know that ex post some consumers will face higher prices at peak times than at off-peak times. I further demonstrate that this pricing strategy is the unique competitive equilibrium in a model with aggregate uncertainty about consumers’ time preferences.

The practice of rationing the availability of lower-priced units is known as yield management (though the more descriptive term “revenue management” is now more commonly used in industry) and is practiced by airlines, hotels, and other industries. While not a general model of yield management, this article presents a stylized model that suggests yield management may be an efficient equilibrium response to uncertainty about the distribution of consumers’ departure time preferences. Yield management can

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efficiently shift some of the stochastic or unanticipated component of firms’ uncertain demands to the off-peak period.  

Because restrictions usually accompany lower prices, industry critics are quick to label yield management as a form of price discrimination, a claim typically supported by citing the large price discrepancy between tourist and business travellers on U.S. airlines that has arisen since deregulation. In defense of their pricing practices, industry insiders point out that the fare differences are even greater in Europe, where most tourists travel on charter flights. Robert Crandall, the CEO of American Airlines, who in 1979 as chief financial officer was largely responsible for the airline’s introduction of both the SuperSaver discount and yield management, wrote the following in an open letter to the company’s customers:

Because the airline business is both extremely complex and widely misunderstood . . . it has generated a number of myths—among them the notion that business travelers subsidize pleasure travelers. . . . To the extent that any market segment can be said to be subsidizing another, it is pleasure travel which is subsidizing business travel—not the other way around.  

Mariott describes a similar reverse cross-subsidization in the hotel industry:

The fact of the matter is, if it weren’t for incremental leisure guests, business guests would have to pay a higher price for their rooms in order for the hotel to meet financial obligations. I’d have to offer all our guests a $79 room, but in order to cover the costs of the hotel and ensure returns to our investors we must differentiate. The bottom line is this: either we accommodate both guests, one paying $79 and one paying $125, or we ask the business guests to pay $145.  

Neither of these descriptions is consistent with price discrimination, but I show that they are consistent with a peak-load pricing model in which firms are uncertain which times are peak. If tourists have weak time preferences and business travellers have strong time preferences, I find that tourists not only consume a relative majority of the low-priced seats, but also that the prices paid by business travellers actually decrease with the number of tourists.  

I extend the static single-period competitive models of uncertain demand first studied by Prescott (1975) and later by Eden (1990), Dana (1993), and, in a model of vertical restraints, Deneckere, Marvel, and Peck (1996). These articles show that if aggregate demand is uncertain, prices are rigid and set before demand is realized, and capacity costs are sunk, then the competitive equilibrium exhibits price dispersion; the lowest-priced units sell with the highest probability while the highest-priced units sell with the lowest probability. Consumers buy at the lowest price still available unless it exceeds their willingness to pay, so the higher-priced units are more likely to be bought by high-valuation consumers despite the fact that firms have no market power and that

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2 One article that examines the efficiency of yield management methods is Botimer (1996).
5 A result with a similar interpretation is presented in Dana (1998). There, consumers with certain demands benefited those with uncertain demands; here, consumers who are willing to change departure times benefit those with more rigid preferences.
6 The article can also be interpreted as adding price dispersion and demand shifting to a model of stochastic peak-load pricing and as adding competition and demand shifting to models of revenue management. Belobaba and Wilson (1996) is one of the few revenue management articles to consider the impact of competition, but like most of this literature it treats prices as exogenous.
consumers are treated identically ex ante. When consumers’ valuations are heterogeneous and rationing is proportional, then price-dispersed equilibria are typically more efficient than uniform prices because low-valuation consumers buy relatively less when demand is high (they are unwilling to pay the high prices that remain after low-priced units have stocked out) and high-valuation customers buy relatively more.

The difference between this article and the earlier ones is that I examine two time periods, introduce demand shifting, and consider heterogeneity in consumers’ costs of switching flights (valuations for consumers’ preferred flight are homogeneous). Also, in the very stylized model that I present, all the aggregate demand uncertainty is over consumers’ preferences for departure times rather than the level of aggregate demand. Absent demand shifting, my simple model is a two-period version of the Prescott model with the curious property that demand for the two periods is perfectly negatively correlated. So it is not surprising that this article shares all the weaknesses of the earlier literature. In particular, it makes the strong assumption that prices are rigid and that demand is allocated by rationing and not market clearing.

The model has a second interpretation. Instead of characterizing departures at different times, the model can be interpreted as characterizing departures from two different airports, such as San Francisco International and Oakland in the Bay Area, or to two different destinations, such as Cancun and Jamaica. Aggregate uncertainty is correspondingly interpreted as uncertainty about geographical preferences as opposed to temporal preferences. In either interpretation, price dispersion can efficiently shift part of the stochastic element of demand from overcrowded flights to less crowded ones, reduce the cost of unutilized capacity, and reduce the fares paid by consumers.

Importantly, I do not try to explain the use restrictions or “fences,” such as Saturday-night stayover or advance purchase requirements, that might segment consumers.7 Gale and Holmes (1992) consider a monopoly model with the same demand structure as mine and show that a monopolist, and a social planner, can use advance purchase discounts to divide uncertain peak demand more evenly between two departures (see also Gale and Holmes, 1993). In their model, spot markets are constrained to have uniform prices, and advance purchase discounts are modelled as a mechanism designed to selectively screen for consumers with weak time preferences before they learn which flight they prefer. Dana (1998) studies advance purchase discounts in a slightly different way, and in competitive markets, by assuming that they are used to selectively screen for consumers with high certainty of demand before they know whether or not they want to fly.8 Like those articles, this article considers a model where early consumers pay lower fares, but unlike them, here there is no distinction in the information available to consumers who buy early at low prices versus those who buy late at high prices.

The article is divided into several sections. Section 2 describes the model. Section 3 derives the competitive equilibrium. Section 4 considers monopoly pricing under the same assumptions. In Section 5 I show that tourist travellers actually subsidize business travellers. Section 6 examines the social planner’s pricing policies under the same assumptions; the competitive equilibrium is not Pareto optimal, but it is superior to uniform pricing. In Section 7 I briefly consider the effect of changing the assumption that low-priced units are allocated randomly to consumers willing to buy them. In

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7 Actually, airline fares are generally listed without respect to departure time, and time-of-day or day-of-week limitations are known as “slot” or “flight” restrictions. These restrictions are implicitly modelled here while others are not.

8 Dana (1998) extends the Prescott model by adding advance purchase discounts and heterogeneous consumer demand uncertainty.

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Section 8 I show that the results are robust to some alternative specifications of the model. Concluding remarks are in Section 9. Proofs are in the Appendix.

2. The model

I assume a finite measure \( N \) of consumers who vary with respect to their \textit{ex post} departure time preferences and with respect to their willingness to substitute their less preferred time for their preferred time. For ease of exposition I imagine that the product is air travel, but the model is intended to be more general. Consumers’ reservation values, \( V \), are identical, but \( w \), the disutility from flying at their least preferred time, is distributed on \([0, \bar{w}]\) with a continuous cumulative probability distribution function \( F(w) \) satisfying \( F(0) = 0 \). Consumers have unit demands with reservation value \( V - w \) for flying at their least preferred departure time and reservation value \( V \) for flying at their most preferred departure time. I assume that \( \bar{w} \) is larger than the cost of capacity to guarantee that some consumers are always willing to bear the entire cost of being served at their most preferred time. I further assume that the distribution of \( w \) is independent of the \textit{ex post} departure time preferences of consumers.

I sometimes refer to \( w \) as a “waiting cost” even though a consumer who wants to fly on the later flight cannot literally “wait” for an earlier flight (though a business traveller who takes an earlier flight may have to “wait” at his destination for his meeting to begin). More generally, the cost \( w \) represents the opportunity cost to the consumer of spending more or less time at his destination (e.g., the cost of leaving a resort early or business trip late).

Firms are uncertain about which of two times, A and B, consumers will prefer to fly.\(^9\) It is common knowledge that consumers’ preferences are correlated and that either time may be the peak period. I assume each time is equally likely to be the peak. I also assume there are only two possible states of demand: either A is the peak flight time and \( N_1 \) consumers prefer to fly at A while \( N_2 < N_1 \) consumers prefer to fly at time B, or B is the peak flight time and \( N_1 \) consumers prefer to fly at B while \( N_2 < N_1 \) consumers prefer to fly at time A. Of course, \( N_1 + N_2 = N \). While \( N_1 \) and \( N_2 \) characterize consumers’ preferences, some consumers may be induced or forced to fly at their less preferred time.

The assumptions capture the idea that there is uncertainty about the relative demand for the two departure times. For simplicity \( N \) is fixed, even though this implies that demand for the two departures is perfectly negatively correlated. However, the results hold as long as the demand for the two departures is not perfectly positively correlated. In particular, they hold when demand for the two flights is independently distributed (see Section 8).

A consumer of type A (a consumer who prefers to fly at time A) receives utility \( U = V - p_A \) if he or she flies at time A and receives utility \( U = V - w - p_B \) if he or she flies at time B. The appearance of the cost \( w \) is reversed for a consumer of type B. When a type-A consumer makes his or her purchase decision he or she will choose the larger of the two utilities (assuming both are available). So if \( p_A \leq p_B + w \), the consumer will choose time A. Although every consumer has some nonnegative waiting cost, \( w \), in equilibrium only passengers who \textit{ex post} prefer to fly at the peak time will ever have to bear that cost.

The cost of producing one seat, which can be used to fly one passenger during each time slot, is \( 2k \), or \( k \) per seat-flight, and the cost of serving a passenger if the seat

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\(^9\) I rule out sales before consumers learn their preferences, which can be justified by assuming consumers learn their own time preferences at the same time they become aware of their demand.
is sold is \( c \) (I assume all sold seats are used, so there are no cancellations or no-shows). The capacity cost \( k \) differs from the marginal cost \( c \) because it is incurred whether or not the seat is sold. Of course, the capacity cost \( k \) could be interpreted as the shadow cost associated with a capacity constraint rather than a direct cost of capacity, and none of the results presented here would be changed. As discussed above, I assume \( \bar{w} > k \).

Firms choose capacity and allocate their capacity over different price levels. Formally, firms’ strategies are chosen from the set of price distributions (quantity measures over a price support). Firms can also be interpreted as offering a single price, with the price dispersion arising because different firms offer different prices.

The remaining assumptions are more restrictive and deserve special attention. Most important, I rule out any price changes after some sales have taken place. If prices and quantities were completely flexible, price dispersion in this model would vanish. However, prices change relatively infrequently in the airline industry (though using yield management software allows firms to change quantities daily). Firms may find it costly to change prices, especially when prices are advertised. Consumers need time to respond to ads and to resolve their travel plans. Business travellers fearing price gouging may be upset if unrestricted ticket prices change from trip to trip. Also, the process of making equilibrating price changes might appear anticompetitive to government agencies. While these arguments can explain some rigidity, clearly a better model would allow some flexibility as well. But it is important to note that airlines currently lack formal mechanisms to compute optimal prices, let alone to determine how optimal prices change dynamically with sales. Even as yield management software becomes more and more sophisticated, it still treats prices as exogenous and in most cases does not even use contemporaneous prices.

I also assume that consumers arrive in the market in random order to make their purchasing decisions. In particular, the order in which consumers arrive is independent of their valuations, their waiting cost \( w \), and their time preferences. Consumer demand is rationed using the proportional, or random, rationing rule as in Carlton’s (1977) model of stochastic peak-load pricing. In their related article, Gale and Holmes (1992) also assumed proportional rationing. Because the incentive to arrive early might not be the same for every consumer, I also briefly consider the parallel rationing rule (considered by Brown and Johnson (1969) in the stochastic peak-load pricing context). I find that only in the unlikely case where consumers are rationed on the basis of their waiting costs and not their valuations is rationing likely to be efficient without price dispersion.

I assume that the prices for both flights are observable at the time of purchase, and consumers make their purchase decisions in advance of their consumption (so that both options are available to them). While this is reasonable for airlines and hotels, this assumption is not currently satisfied for consumer phone service or electric power. When the transactions costs of communicating prices are high relative to the value of the product or service, firms will instead turn to other pricing methods, such as uniform stochastic peak-load prices.

Finally, I assume that firms are unable to write any type of contingent forward contracts with their customers. The priority service pricing literature has demonstrated

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10 Most revenue management programs require only information about the relative prices, which do not fluctuate significantly as the price levels, so historical averages are usually sufficient.

11 I am assuming consumers with different time preferences are given the same set of choices. However, in a competitive equilibrium firms have no market power with which to exploit knowledge of consumers’ types. If firms could restrict seats to only low-waiting-cost or only high-waiting-cost consumers, they would not benefit from doing so.
that even very simple forward contracts can achieve efficient allocations under demand uncertainty (see Harris and Raviv, 1981, Chao and Wilson, 1987, and Wilson, 1989) so it is obviously important to understand why contingent contracts are not used in the airline industry. One possible answer is that consumers make other travel-specific investments or reliance expenditures well before their trips, so the consumers’ transactions costs associated with contingent contracts are quite high. Another is that the costs to the firm of performing last-minute allocations and communicating the results to consumers are too high. Nevertheless, for comparison I derive and discuss the optimal forward contract prices in Section 6.

3. Competitive equilibrium pricing

A competitive equilibrium is defined as in Prescott (1975), Eden (1990), and Dana (1993). Since firms are free to offer units at any price and prices do not clear the market, I need to be explicit about how I define the terms of trade. I assume that each firm takes as given the probability of sale associated with every possible price offer. So firms see the “announced” equilibrium prices and the “announced” equilibrium probabilities and maximize profits given those prices and probabilities. Of course, the probability that any unit will sell in equilibrium is endogenous and depends on the number of units offered at the same and lower prices, but in the spirit of perfect competition, I assume that firms do not think they are large enough to affect these probabilities.

Firms’ strategies (for each time slot) are schedules of prices and quantities, or, more formally, quantity measures on the support of nonnegative prices. These price distributions are represented by the cumulative distribution functions $Q_i(p)$, where $Q(p) = \Sigma_i Q_i(p)$ is the aggregate or industry distribution. Given the specification of demand uncertainty, the probability of sale, denoted $\varphi(p)$, is uniquely determined by $Q(p)$.

Definition. A competitive equilibrium is a distribution of prices, $Q(p) = \Sigma_i Q_i(p)$, and an associated probability-of-sale function, $\varphi(p)$, such that (i) given $Q(p)$, if consumers arrive in random order and make utility-maximizing purchase decisions given the lowest price available on each flight, then the probability that a unit at price $p$ sells in equilibrium is $\varphi(p)$, and (ii) given $\varphi(p)$, each firm’s price distribution $Q_i(p)$ is consistent with profit maximization, that is, $(p - c)\varphi(p) - k \leq 0$, and if the price distribution $Q(p)$ has positive measure on any interval containing $p$, then $(p - c)\varphi(p) - k = 0$.

This definition requires that firms have rational beliefs about the probability that their units will sell, that they act as if they cannot influence those probabilities, and that profits are zero at every equilibrium price.

In the competitive equilibrium of this model, firms will offer at most two prices. Given two equally likely demand states, there will be one price cutoff below which every seat sells with probability one and another higher-price cutoff below which every seat sells with probability one-half (even higher prices will be associated with zero probability of sale), yet in a zero-profit equilibrium all seats sold with the same probability must carry the same price, so all firms will price at these two cutoffs. Under

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12 For example, airlines might sell high-priority tickets and low-priority tickets, where ex post, after all sales are made the high-priority ticket holders are allowed to choose their departure time preference freely while low-priority ticket holders are allocated departure times subject to availability.

13 In Dana (1993), I show that under reasonable conditions the competitive equilibrium (in a single-period model with uncertain demand) derived using this definition is the limit of the equilibrium of the analogous oligopoly model as the number of firms approaches infinity.
proportional, or random, rationing I show in Proposition 1 that firms will offer exactly
two prices, \( p_L = c + k \) and \( p_H = c + 2k \).

The equilibrium quantities depend on the magnitude of the peak and the distribution
of \( w \). Let \( Q_L \) be the total number of units offered at price \( p_L \) at each time slot
and let \( Q_H \) be the total number of seats offered at price \( p_H \) at each time slot. Since at
least \( N_2 \) consumers are sure to want to travel at time A (and another \( N_2 \) consumers are
sure to want to travel at time B), it follows that \( Q_L \geq N_2 \); firms know that at least \( N_2 \)
will be sold, since these consumers will always prefer to pay \( p_L = c + k \) at their
preferred time to anything offered at the other time slot.

At the peak time, only \( Q_L \) consumers can be served at a price \( p_L = c + k \), but
another \( N_1 - Q_L \) consumers prefer to fly at the peak time. What if all these consumers
chose to fly at their preferred time? Then all the additional seats offered would have
to be priced at \( p_H = c + 2k \), since these seats would sell only half of the time (the
seats are empty on the off-peak flight). And sales on the off-peak time would be limited
to \( N_2 \). Note that this is the equilibrium when \( F(k) = 0 \), and it is identical to the Prescott
equilibrium; the equilibrium quantities are \( Q_L = N_2 \) and \( Q_H = N_1 - N_2 \).

However, this is not a competitive equilibrium in general, since some consumers
will be willing to fly at their less preferred time in order to save \( p_H - p_L = k \). Because
of demand shifting, firms can offer more units at price \( p_L \) at each time slot and still be
assured that they will sell with probability one. In equilibrium,

\[
Q_L = N_2 + F(k)(N_1 - Q_L) \tag{1}
\]

and

\[
Q_H = (1 - F(k))(N_1 - Q_L), \tag{2}
\]

so

\[
Q_L = N_2 + (N_1 - N_2)\left( \frac{F(k)}{1 + F(k)} \right) \tag{3}
\]

and

\[
Q_H = (N_1 - N_2)\left( \frac{1 - F(k)}{1 + F(k)} \right) \tag{4}
\]

are the competitive equilibrium quantities. Note that these quantities satisfy

\[
Q_H + 2Q_L = N_1 + N_2
\]

while total capacity is \( Q_H + Q_L \) (on each flight); capacity \( Q_H \) is unutilized in equilib-
rium.

The competitive equilibrium is characterized in Proposition 1.

**Proposition 1.** If \( V > c + 2k \), then there exists a unique industry competitive equilib-
rium price distribution in which \( Q_L \) units are offered at a price \( p_L = c + k \) and \( Q_H \)
units are offered at a price \( p_H = c + 2k \) for each time slot, where the quantities \( Q_L \)
and \( Q_H \) are given by (3) and (4).

It should be clear that the assumption of rectangular demand is inessential to this
result. If consumers had heterogeneous valuations, then the equilibrium prices would
Proposition 1 describes only the industry price distribution. Because of the constant-returns-to-scale assumption, firms’ capacities are not determined in equilibrium. But more important, neither is the mix of high and low prices that a firm offers. Both a symmetric equilibrium where each firm offers the same mix of low and high prices and an asymmetric equilibrium in which firms specialize by charging either high or low prices are consistent with the model.

To illustrate the equilibrium, consider the following simple numerical example: \( N_1 = 120, N_2 = 60, \ c = 20, \ k = 100, \) and \( F \) is the cumulative uniform distribution with \( w = 250. \) The competitive equilibrium prices are \( p_L = 120 \) and \( p_H = 220. \) Firms supply \( Q_L = 70 \) units of capacity at the low price and \( Q_H = 40 \) units of capacity at the high price. Total capacity is 110, which is less than \( N_1 \) because some consumers choose to fly at their less preferred time. If A turns out to be the peak flight, the first 105 consumers select their flight solely on the basis of their time preferences (70 fly peak, 35 off-peak), while the next 75 consumers choose on the basis of the strength of their preference and the difference between the fares still available on the two flights. As a consequence, only one-third of the first 105 consumers choose to fly off-peak, while 41.67 of the next 75 consumers fly off-peak (16.67 consumers despite the fact that they prefer to fly at the peak time).

4. Monopoly pricing

In this section I present a brief and somewhat informal description of the monopoly problem. A monopolist could charge \( V \) and extract all of the surplus from the consumers, but if he did so he would have to provide \( N_1 \) units of capacity for each departure, resulting in significant excess capacity for the off-peak flight. Alternatively, the monopolist could set two prices for each flight in hopes that a few lower-priced seats on the off-peak flight would induce some passengers to switch flights. The cost is that the firm forgoes some revenue on both its flights, since both must be discounted \( \text{ex ante} \) (the firm does not know which flight will be peak when it sets its price). The benefit is that the firm saves \( 2k \) for each net passenger who switches to the off-peak flight.

Suppose the monopolist offers at most two prices on each flight, \( p_L \) and \( p_H \) (in fact, more than two may be optimal). I assume that the monopolist offers enough low-priced units so that it does not need to ration them at the off-peak time. That is, there is enough output produced so that every consumer has the option to fly off-peak at \( p_L \) or peak at \( p_H; \) of course, the peak seats offered at \( p_L \) will be rationed. This is intuitive—since consumers arrive at random, if the monopolist wants to offer an incentive to induce some consumers to switch, then he will want to offer it to them all.

Recall that \( F(w) \) is the cumulative distribution of waiting costs. I also assume that \( f(w)/(1 + F(w))^2 \) is weakly decreasing. Under these assumptions the monopolist’s profit function is \( 2Q_L(p_L - c - k) + Q_H(p_H - c - 2k). \) However, it is clear that the monopolist will set \( p_H = V. \)

Letting \( d = p_H - p_L, \)

\[
Q_L = N_2 + F(d)(N_1 - Q_L) \tag{5}
\]

and

\[
Q_H = (1 - F(d))(N_1 - Q_L). \tag{6}
\]
Solving these two equations yields

\[ Q_L = N_2 + (N_1 - N_2) \left( \frac{F(d)}{1 + F(d)} \right) \] (7)

and

\[ Q_H = (N_1 - N_2) \left( \frac{1 - F(d)}{1 + F(d)} \right). \] (8)

So the monopolist’s objective function can be written

\[
\max_{d \geq 0} 2 \left( N_2 + (N_1 - N_2) \left( \frac{F(d)}{1 + F(d)} \right) \right) (V - d - c - k) \\
+ (N_1 - N_2) \left( \frac{1 - F(d)}{1 + F(d)} \right) (V - c - 2k)
\] (9)

subject to \( d \geq 0 \). The first-order condition is

\[
-2 \left( N_2 + (N_1 - N_2) \left( \frac{F(d)}{1 + F(d)} \right) \right) + 2 \frac{f(d)(N_1 - N_2)}{(1 + F(d))^2} (k - d) = 0.
\] (10)

Evaluating the left side of this equation at \( d = 0 \) yields

\[
-N_2 + f(0)(N_1 - N_2)k,
\] (11)

which is clearly positive when \( N_2 \) is close to zero, so the monopolist will offer discount prices whenever \( N_2 \) is small.

If \( d = 0 \), the monopolist offers \( Q_H + Q_L = N_1 \) seats at the uniform price \( p_H = p_L = V \). Unless \( N_1 \) is small, the price discount is not worth it; too many consumers flying off-peak get the discount without having to switch. The cost of capacity also matters, since every peak seat shifted reduces costs by \( 2k \). Intuitively, at a cost of \( 2N_2\Delta p \) lost revenue for discounting \( N_2 \) seats at each time, the monopolist persuades \( f(0)(N_1 - N_2)\Delta p \) consumers to fly at the off-peak time, saving \( 2k \) per consumer. In general, the decision to offer a discount depends on how many consumers have low waiting costs and on how uncertain the demand is. It also depends on the cost of rationing seats, since nonprice rationing is in general an alternative strategy for the firm (though not in this example, because no consumer will pay \( V \) to fly at the least preferred time). However, while a monopolist may or may not offer discount seats, competitive firms always offer discounts.

Note that when \( d = k \) the left-hand side of (10) is negative, so \( d < k \). So the monopolist shifts less demand from the peak time to the off-peak time than competitive firms do. Price dispersion in the monopoly case is actually smaller than in the case of competition. This result is consistent with Dana (1993), where the degree of price dispersion caused by price rigidities and costly capacity actually increases with the number of firms.\(^{14}\) It is also empirically consistent with the airline industry (see Bor-\(\text{en}e\text{nstein and Rose, 1994).}

\[^{14}\text{It is also consistent with the literature on advance purchase discounts. Price discounts can increase with competition (see Gale, 1993).}\]
5. Tourist versus business travellers: who subsidizes whom?

The model here has striking, but intuitive, implications about the effect of consumer mix on the equilibrium prices. Suppose that tourists differ from business travellers only in their disutility of flying at their least preferred time but have the same ordinal ranking of departure times. Define all consumers with \( w < k \) to be “tourists” and all consumers with \( w \geq k \) to be “business travellers,” and suppose that the departure time preferences of all travellers are perfectly correlated (i.e., \( N_5 = 0 \)). Note that the social cost of serving business travellers in the absence of tourists is \( c + 2k \) per consumer because half of the capacity goes idle, while the social cost of serving tourists in the absence of business travellers is \( c + k + E[w | w \leq k]/2 \) because half of the tourist demand can be shifted to utilize the off-peak capacity at cost to those consumers of \( E[w | w \leq k] \). In a competitive equilibrium, output is offered at two prices, \( p_H = c + 2k \) and \( p_L = c + k \), and the expected price paid by business travellers is thus strictly less than \( c + 2k \). Tourist travellers all purchase at the price \( p_L = c + k \), but in equilibrium more than half of them end up flying at their least preferred time. An increase in the relative number of tourist travellers benefits business travellers while it makes tourist travellers worse off.

Formally, if there are \( T \) tourists and \( B \) business travellers, then from (3) and (4) the equilibrium quantities are

\[
Q_L = (B + T) \left( \frac{T}{B + 2T} \right) \tag{12}
\]

and

\[
Q_H = (B + T) \left( \frac{B}{B + 2T} \right), \tag{13}
\]

and the expected price paid by business travellers is

\[
E[p] = \left( \frac{B + T}{B + 2T} \right)\left( c + 2k \right) + \left( \frac{T}{B + 2T} \right)\left( c + k \right) \tag{14}
\]

So the expected price paid by business travellers is decreasing in \( T \), increasing in \( B \), and equals \( c + 2k \) when \( T = 0 \), while their expected utility \( V - E[p] \) is increasing in \( T \). Since the fraction of tourists who fly at their least preferred time is \( (B + T)/(B + 2T) > \tfrac{1}{2} \), tourists’ expected utility is

\[
V - \left( \frac{B + T}{B + 2T} \right)\left( c + k + w \right) - \left( \frac{T}{B + 2T} \right)\left( c + k \right), \tag{15}
\]

so while the price tourists pay is unchanged, tourists’ utility is decreasing in \( B \), increasing in \( T \), and is maximized when \( B = 0 \).15

In models where aggregate demand is uncertain and prices do not clear markets, the price a consumer pays depends on a variety of the characteristics of other consumers’ demands, even without economies of scale or scope. The question “who subsidizes

15 The results do not depend on the assumption of a uniform reservation value. Even if consumers’ reservation values are systematically higher for business travellers, as is typically assumed in models of price discrimination, the equilibrium prices are the same, though the quantities may be different.

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wom" has many answers. Here I show that under proportional rationing, an increase in the number of consumers with weak departure-time preferences benefits consumers who are less flexible and have strong departure-time preferences. In Dana (1998) I show that under proportional rationing, an increase in the number of consumers with more certain demands benefits consumers with more uncertain demands, and that under parallel rationing, an increase in the number of consumers with low valuation benefits consumers with high valuation (but advance purchase discounts may eliminate these effects by segmenting the two types). Each of these characteristics is obviously a real component of the difference between a typical tourist traveller and a business traveller, but it remains an empirical matter to attempt to sort among these effects.

6. Optimal pricing

- First best. The first-best allocation is obtained with peak-load prices when the planner sets prices after demand is known, or with the use of complete contingent forward contracts (Arrow-Debreu securities). The equilibrium spot prices are

\[ p_{\text{peak}} = c + 2k \quad \text{and} \quad p_{\text{off-peak}} = c \]

if \( N_1(1 - F(2k)) > N_2 + F(2k)N_1 \), so there is excess capacity off-peak, and

\[ p_{\text{peak}} = c + k + d/2 \quad \text{and} \quad p_{\text{off-peak}} = c + k - d/2 \]otherwise,

where \( d \) is defined by \( N_1(1 - F(d)) = N_2 + F(d)N_1 \). The equivalent Arrow-Debreu state-contingent prices (assuming that consumers know the strength of their preferences, but not their preferred departure time, when they purchase) are half of these prices, since each state is equally likely. At the peak time the marginal consumer is served only if his or her waiting cost exceeds \( 2k \), the marginal cost of capacity. The allocation is efficient because it minimizes the sum of firms’ production costs and consumers’ waiting costs.

Clearly the competitive equilibrium is inefficient relative to the first-best allocation; in the competitive equilibrium some passengers fly at the peak time even though their willingness-to-pay for a peak seat may be very small and there is excess capacity on the off-peak flight.

- Uniform pricing. Next I compare the competitive equilibrium to the second-best allocation obtained when the planner is restricted to uniform prices. With a uniform price, consumers will always fly at their preferred time unless they are rationed. At the welfare-maximizing uniform price, capacity is rationed only if \( E[w] \leq 2k \). If \( E[w] > 2k \) (which I call case 1), then the optimal policy is to satisfy every consumer’s time preferences. But if \( E[w] \leq 2k \) (which I call case 2), only those consumers who happen to arrive early will have the choice of flying at the peak time.

In case 1 the planner maximizes welfare by supplying every consumer who wants to fly at the peak time, so capacity will be \( N_1 \). Rationing seats is inefficient, since the marginal
consumer has an expected waiting cost that exceeds the social cost of an additional seat (recall that two seats, one on each flight, need to be provided to service an additional peak consumer). The zero-profit price is \( \hat{p} = c + k / \phi \), where \( \phi = (N_1 + N_2) / 2N \) is the probability that an average seat will be sold and the equilibrium quantity on each flight is \( Q = N \). No demand is shifted. The competitive equilibrium with price dispersion is clearly more efficient in this case, because in it each customer induced to change departures in equilibrium suffers a waiting cost no more than \( k \) while firms’ production costs fall by \( 2k \). Here, with uniform prices, consumers’ waiting costs are zero, but firms hold too much capacity.

When \( E[w] \leq 2k \), the planner shifts demand by rationing the number of units offered at the peak time. However, because consumers arrive in random order, those who are forced to fly off-peak have no weaker preferences for departure times than those whose preferences are satisfied. The optimal quantity is \( Q = (N_1 + N_2) / 2 \), the zero-profit price is \( \hat{p} = c + k \), and off-peak capacity is fully utilized. Because the expected waiting cost of the marginal consumer who is rationed is the same as the waiting cost of the consumer who is not rationed, the allocation of consumers is inefficient.

Unlike case 1, where price dispersion increases capacity costs (because it increases unutilized capacity) and lowers waiting costs (because both high- and low-waiting-cost consumers all fly at their preferred time), in case 2 price dispersion lowers capacity costs and raises waiting costs. In this case the competitive equilibrium with price dispersion is not necessarily more efficient than uniform pricing. It is possible that the additional costs imposed by consumers who buy high-priced seats (in the competitive equilibrium) exceed the benefits.

**Proposition 2.** If \( F(k) = 1 \), then the optimal uniform price (case 2) and competitive equilibrium are equivalent. If \( F(k) < 1 \) and \( E[w] < 2k \) (case 2), then the difference between social welfare (expected capacity cost plus waiting costs) under the competitive equilibrium and social welfare under optimal uniform pricing is

\[
(Q_H/2)[2E[w|w > k] - 2k - E[w]],
\]

which may be positive or negative. And finally, if \( F(k) < 1 \) and \( E[w] \geq 2k \) (case 1), then social welfare under the competitive equilibrium exceeds social welfare under optimal uniform pricing by \( (Q_L - N_2)(2k + E[w|w < k]) \), which is strictly positive as long as \( F(k) > 0 \).

The proposition clearly implies that \( E[w|w > k] > 2k \) is sufficient for there to be no gain from imposing uniform pricing. In fact, with a uniform distribution, \( w \sim U[0, \bar{w}] \), I can show that the gains from equilibrium price dispersion are positive as long as \( E[w|w > k] > (1/2)k \), or equivalently \( \bar{w} > 2k \), so \( E[w|w > k] > 2k \) is clearly a weak sufficient condition. But it is quite intuitive because it implies that if the customers who benefit directly from price dispersion (the high-valuation customers who would have flown at their least preferred time) are willing to pay for the marginal high-priced seat (at cost \( 2k \)), then the total surplus is increased. The efficiency gain is even larger because the high-valuation passenger’s move from the off-peak departure to a high-priced seat on the peak departure leaves a seat vacant that is then used by low-valuation consumers who would have flown peak. In this way the supply of low-priced seats can be reduced for a net savings of \( k \) less the low-valuation customer’s expected waiting cost.

In the numerical example I considered earlier \( E[w] < 2k \), so at the optimal uniform price demand is rationed (case 2). The uniform price is 120 and price dispersion is...
more efficient than uniform pricing. The average reduction in waiting costs relative to uniform pricing is 20.83 per consumer, while the average increase in capacity costs is 16.67 per consumer (though the expected gains are only positive for the high-valuation consumers). In the same example, if \( \bar{w} \in (0, 100) \), then the competitive equilibrium is equivalent to optimal uniform pricing. If \( \bar{w} \in (100, 200) \), then the competitive equilibrium exhibits price dispersion, but optimal uniform pricing is more efficient than equilibrium pricing. If \( \bar{w} \in (200, 400) \), then the competitive equilibrium exhibits price dispersion and is more efficient than uniform pricing. Finally if \( \bar{w} > 400 \), then demand is not rationed at the optimal uniform price (case 1), and price dispersion is still more efficient than optimal uniform pricing.

\[\square\] **Dispersed pricing with a zero-profit constraint.** Can a social planner do better than competitive firms if it is free to set dispersed prices? In the competitive equilibrium, if the social planner could persuade the marginal consumer to fly off-peak instead of peak, then everyone could be made better off. The savings in capacity costs would be \( 2k \), one fewer seat is offered on each flight, and the social cost is only \( k \) (the consumer at the margin between choosing the peak and the off-peak time has a waiting cost \( w = k \)).

But if the planner faces a zero-profit constraint \( (p - c)\varphi(p) - k \leq 0 \) (with strict equality holding whenever the price distribution has positive measure), then the planner’s optimal pricing and allocation are the same as the competitive equilibrium. Indeed, the welfare-maximizing uniform price, which might be more efficient than the price-dispersed competitive equilibrium (when \( F(k) < 1 \)), does not satisfy the zero-profit constraint (profits are positive at the price \( c + 2k \)), so the planner can only implement the competitive equilibrium. This follows from Proposition 1, which shows that there is a unique allocation that satisfies the zero-profit condition.

**Proposition 3.** If the social planner faces the zero-profit constraint \( (p - c)\varphi(p) - k \leq 0 \) with strict equality holding whenever the price distribution has positive measure, then the planner maximizes consumer plus producer surplus by implementing the competitive equilibrium: the planner sets two prices, \( p_L = c + k \) and \( p_H = c + 2k \), and chooses quantities given by (3) and (4).

**Proof.** Follows immediately from Proposition 1.

\[\square\] **Dispersed pricing with a nonnegative profit constraint.** When the planner is free to set prices so that firms earn strictly positive profits on some sales, she can do better than the competitive equilibrium (and uniform pricing) by raising the price of the high-priced seats strictly above the competitive, zero-profit level. This has two effects. First, it increases the incentive for consumers who face different prices to fly off-peak instead of peak. Second, it increases the number of low-priced seats available at both times and hence reduces the number of consumers who face different prices for the two times. However, starting at the competitive equilibrium, the net effect favors increasing the price of the high-priced seats.

Since I want only to show that the social planner can do better with dispersed prices than the competitive equilibrium, I make the restrictive assumption that the planner sets two prices, \( p_L \) and \( p_H \), and offers only two classes of seats, one that sells with probability one and one that sells with probability \( \frac{1}{2} \). Since profits must be nonnegative on each unit (and since the level of prices does not affect welfare), I assume that the planner chooses \( p_L = c + k \), the lowest price acceptable to producing firms.\(^{16}\)

\(^{16}\) The level of prices does not affect demand or welfare, since I have assumed that \( V \) is large and demand is inelastic; however, the difference in prices does affect demand for peak versus off-peak flights.
I then find \( p_H \), or equivalently the difference in the prices, \( d = p_H - p_L \). Given \( d \), the equilibrium quantities can be derived as in (3) and (4):

\[
Q_L = N_2 + (N_1 - N_2) \left( \frac{F(d)}{1 + F(d)} \right) \quad (16)
\]

and

\[
Q_H = (N_1 - N_2) \left( \frac{1 - F(d)}{1 + F(d)} \right). \quad (17)
\]

The social planner’s optimization is

\[
\max_d (N_1 + N_2)(V - c) - 2(Q_L + Q_H)k - \frac{F(d)}{1 + F(d)} \frac{(N_1 - N_2) \int_0^d wf(w) \, dw}{F(d)} \quad (18)
\]

or, using (16) and (17),

\[
\max_d (N_1 + N_2)(V - c) - 2k \left[ N_2 + \frac{(N_1 - N_2)}{1 + F(d)} \right] - \frac{F(d)}{1 + F(d)} \frac{(N_1 - N_2) \int_0^d wf(w) \, dw}{F(d)}. \quad (19)
\]

The first-order condition yields

\[
[1 + F(d)]d - F(d) \int_0^d wf(w) \, dw = 2k. \quad (20)
\]

The marginal benefit of eliminating one seat on each plane is \( 2k \), while the marginal cost of eliminating the seat is that one marginal consumer will have to wait for an off-peak flight, and since the number of low-priced seats is also affected, \( F(d) \) consumers who would have obtained low-priced seats will be stocked out and choose to wait for an off-peak flight. Equation (20) implies \( k < d < 2k \). So when the social planner faces only a nonnegative profit constraint, she can increase consumer plus producer surplus relative to the competitive equilibrium by raising the price of high-priced seats. However, since \( k < d < 2k \), the planner’s allocation is still inefficient relative to the first best.

Most important, this shows that price dispersion is always optimal for the planner. Uniform pricing is feasible here, but two prices are always more efficient. Of course if there are no consumers whose waiting costs exceed \( d \) (that is, \( F(d) = 1 \)), then consumption at \( p_H \) is zero and the allocation is equivalent to uniform pricing (case 2). But as long as there are some consumers whose waiting costs are greater than \( d \) (so it is sufficient that some have waiting cost greater than \( 2k \)), then the optimum exhibits price dispersion.

The planner can do even better by utilizing more price levels. By choosing more prices the planner increases the number of peak consumers who \emph{ex post} face a price
differential between the peak and off-peak flights. In fact, if $V$ were infinite and demand were still inelastic, then the planner's allocation would approach the first best.

7. Parallel rationing

- In this section I briefly explore equilibrium pricing when the order of customer arrival is not random but depends on consumers' preferences. The parallel rationing rule posits that consumers are served in decreasing order of their valuations, and it is the most commonly considered alternative to proportional rationing. But parallel rationing is not well defined in this model because consumers' valuations are represented by both $V$ and $V - w$, and arguably $V + w$ is closest to what one intuitively means by the strength of a consumer's preferences.

  Since all consumers have the same valuations $V$, if rationing depends on $V$ but not $w$ the competitive equilibrium is unchanged. More important, even if consumers had heterogeneous valuations the equilibrium allocation is unchanged. If consumers are served in order of their valuations and not the strength of their time preferences, then parallel rationing does not allocate consumers any more efficiently than random rationing.\(^{17}\)

  But if consumers are rationed on the basis of their departure time preferences, so that those with the highest $w$ buy first, then the allocation is efficient. Intuitively, if those who care most about being forced to fly at their least preferred time make the biggest effort to be early, then rationing is more efficient and may even reach the first best. In particular, if $F^{-1}(N_1/N + (1/2)) \leq k$, then capacity is $N/2$ on each flight, all seats sell for $c + k$, and the off-peak flight is filled to capacity with consumers whose consumption of the peak flight is rationed and the allocation is the first best. If $F^{-1}(N_1/N + (1/2)) > k$, then some seats sell for $c + k$ and others for $c + 2k$, capacity is $N_1(1 - F(k))$, and only $N_2 + N_1F(k)$ units sell on the off-peak flight. The latter allocation is not Pareto optimal because the marginal peak consumer faces a price differential $k$ between the peak and off-peak flights as opposed to $2k$, which is the cost of the marginal unit of capacity. But it is more efficient than the allocation under proportional rationing.

  In a more general model where the level of aggregate demand is also uncertain and consumer demand is elastic, this rationing rule is no longer as efficient or compelling. A more plausible interpretation of the parallel rationing rule would specify that those with both high $V$ and high $w$ would be first, since they are the consumers with the most at stake. But parallel rationing is not efficient in this model because of the possibility that it results in underutilization of capacity.

  But this rationing has two significant shortcomings. First, it is inconsistent with the casual empirical observation in airline markets that consumers with stronger departure time preferences are those generally least likely to buy in advance. Second, when consumers have heterogeneous valuations and heterogeneous waiting costs, then parallel rationing is efficient only if rationing occurs in order of waiting costs $w$ and is independent of $V$. But the strength of a consumer's preferences is more likely to be measured by the combined valuation $V + w$. Rationing in order of valuations is inefficient because of the possibility that it results in underutilization of capacity.

\(^{17}\) The equilibrium is unchanged as long as the distributions of $V$ and $w$ are independent and the lower bound on consumers' valuations satisfies $V > c + 2k$. If $V < c + 2k < V$, parallel rationing may be even worse because a low-valuation consumer may face a price that exceeds his valuation even when his valuation exceeds marginal cost.
8. Extensions

- In this section I briefly consider three important extensions of the model.

- Asymmetric demand distributions. When one period is more likely to be the peak, or even when one period is clearly the peak time but there is uncertainty about how much greater the demand will be at the peak time, the equilibrium prices will no longer be symmetric, but they will still be nonuniform. When either period can be the peak, in equilibrium there will be both low- and high-priced seats offered at each period, and the high-priced seats will sell only if that period happens to be peak. The four prices depend on parameters of the distribution, and the period that is more likely to be peak, or has a more severe peak, will have higher prices. In equilibrium the shadow costs of capacity for each period satisfy $\lambda_A + \lambda_B = 2k$. As long as there is a chance that each period will be the peak, the shadow costs, $\lambda_A$ and $\lambda_B$, are strictly positive and the equilibrium prices are $c + \lambda_A$ and $c + 2\lambda_A$ for time A and $c + \lambda_B$ and $c + 2\lambda_B$ for time B.

  If one period, say time B, is the off-peak time in both demand states but the peak problem is more severe in state 1, the equilibrium prices will still be of the same form, namely, $c + \lambda_A$ and $c + 2\lambda_A$ for time A and $c + \lambda_B$ and $c + 2\lambda_B$ for time B, but now $\lambda_B$ may be zero. To see this, imagine that consumers’ time preferences all satisfy $w > 4k$. Then in equilibrium, all time B seats sell for marginal cost, $c$, and the only consumers to buy at time B are those who prefer time B. In general, too much demand is shifted in state 2 (when capacity at time A is not fully utilized) and not enough in state 1 (when some consumers with valuations between $c + 2k$ and $c + 4k$ fly at time A).

  If $\lambda_B > 0$, then there is price dispersion at the off-peak time even if it is off-peak in every demand state. This occurs when enough consumers have weak time preferences to ensure that off-peak consumption in state 2 is the same as the peak consumption in state 1. As long as an additional unit of capacity at the off-peak time can be utilized in some states to relieve capacity at the peak time (the marginal consumer’s waiting cost is less than $4k$), then the shadow cost of off-peak capacity is positive and prices are dispersed.

  While this suggests that we would not expect to see very high fares on flights that are known to be off-peak, the presence of high fares on these flights may be consistent with this type of model when you consider that many a full-fare ticket entitles the passenger to fly on any flight, including peak flights, as long as seats are still available. Because business travellers require additional flexibility after they have purchased their ticket, airlines allow ticket holders in some fare classes to easily reschedule and severely restrict other discount fare ticket holders from rescheduling or returning their tickets. Courtyard and Hao (1998) consider a monopoly model where the airline screens customers based on their willingness to give up the right to a refund. One interpretation of their model is that airlines sell tickets that are differentiated by the degree of flexibility allowed the traveller.

- Geographic or spatial differentiation. The model has a natural interpretation in terms of geographically differentiated markets as opposed to temporally differentiated markets. When demand is uncertain at two airports that are substitutes for each other, price dispersion will induce efficient reallocation of demand as well. In this case, however, there is no justification for assuming capacities are equal across the two periods. If the costs are symmetric, then the shadow costs of capacity at each site will equal the long-run capacity cost. So there is no reason to think the airport with smaller demand will have lower prices or be a net recipient of passengers who choose their
less preferred airport (or are rationed). The same occurs when two resort destinations are substitutes for one another: if there is uncertainty about which destination will be more popular, then price dispersion allocates demand across the two destinations more efficiently than uniform price rationing.

Another important difference with geographic differentiation is that it is much more difficult to imagine that demands at the two airports are negatively correlated. However, I shall now show that similar results hold when demands are independently distributed.

Combining aggregate demand uncertainty and time preference uncertainty. What happens when demands for the two flights are independently distributed? More generally, what happens when there is both aggregate demand uncertainty, as studied by Prescott and others, and time preference uncertainty, as studied here?

Consider the following four demand-state models representing a combination of these two elements. Aggregate demand can be either high or low, and aggregate time preferences can be stronger either for time A or for time B, creating four different demand states which I denote by \{LA, LB, HA, HB\}. I assume that the aggregate level of demand and time preferences are independently distributed, and I again impose symmetry, so, denoting demand for time \(j\) in state \(i\) by \(N^i_j\), (1) \(N^A_{LA} = N^B_{LB}\) and \(N^A_{LA} = N^B_{LB}\); (2) \(N^A_{HA} + N^B_{LA} = N^A_{LB} + N^B_{HB} = N^H\) and \(N^A_{LA} + N^B_{HA} = N^A_{LB} + N^B_{HB} = N^L\); and (3) the probability of each state is \(\frac{1}{4}\). And because the distributions are assumed to be independent, \(N^A_{LA}N^H = N^A_{LB}N^L\).

Demand for the two times may be either positively, independently, or negatively correlated depending on whether uncertainty about the level of demand or the time preferences contributes more to aggregate demand uncertainty. In a competitive equilibrium, if \(V > c + 4k\), there will be four equilibrium prices offered on each flight, \(c + k, c + (\frac{2}{3})k, c + 2k\), and \(c + 4k\), each with a different probability of sale. Some seats will sell even if the flight time is unpopular and demand is low, \((c + k)\). Others will sell if either it is the more popular flight or demand is high, \((c + (\frac{2}{3})k)\), and some will sell only if it is both \((c + 4k)\). Other units priced at \(c + 2k\) might sell when the flight is more popular and demand is either low or high, or they might sell only when demand is high and the flight is either less popular or more popular; which occurs depends on which factor influences demand more \((N^A_{LA} > N^B_{HB} \text{ or } N^A_{LA} < N^B_{HB})\). Demand shifting occurs because once the cheapest seats at the \textit{ex post} peak time are sold out, consumers will face a price differential between the lowest remaining price for the two times.

Clearly, uniform prices would allocate less efficiently in the high-demand states (though potentially more efficiently in low-demand states). However, price dispersion in this model is being driven by both aggregate uncertainty about the level of demand and aggregate uncertainty about time preferences. Even if there were no uncertainty about time preferences, there would be two equilibrium prices, \(c + k\) and \(c + 2k\).

9. Conclusions

- Uniform pricing is inefficient because of the need to allocate uncertain demand across different departures, even when peak-load pricing is allowed. When firms face forecastable demand fluctuations, traditionally peak-load pricing is both effective and encouraged, but this article shows that when firms face unforecastable demand fluctuations, equilibrium price dispersion can efficiently shift demand and lower capacity costs, particularly when the costs to some consumers of changing departures is high. Equilibrium prices are not optimal (unless prices must satisfy a zero-profit constraint),

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but I show that a social planner will always use price dispersion to allocate more efficiently.

The model also serves as another warning against the use of price dispersion or yield management as evidence of price discrimination or market power. Since efficient management practices can explain many of the practices attributed to price discrimination, policy makers should consider other evidence of market power before concluding that behavior that appears to be discriminatory is anticompetitive. This article gives even stronger evidence of the efficiencies associated with price dispersion and the potential costs of eliminating it.

However, empirically distinguishing these results from the alternative hypothesis that price dispersion is due to market power and price discrimination is not easy (see Borenstein and Rose, 1994, and Shepard, 1991). Moreover, there are economies of scale and scope that could arise from pooling consumer classes that would make it even more difficult to estimate the causes of equilibrium price differences between consumers.

The model could also be extended to explain the practice of overbooking by airlines (in particular in a more realistic model with repeated departures), even when airlines expect to have to pay passengers to wait for a later flight. If firms have the option to pay consumers to switch departure times after they have made their purchases, then they effectively have a price that is contingent on the realization of demand, so it would be surprising if firms could not improve profits by implementing such prices. The only problem is that it must remain individually rational for consumers who prefer to fly at the off-peak time to continue to fly off-peak rather than choose the peak flight in order to extract the reward for being bumped, otherwise these consumers would all choose to fly at the peak time. If consumers’ ex post information about which period will have peak demand is limited to observable prices, then this problem will probably be small.

Appendix

Proofs of Propositions 1 and 2 follow.

Proof of Proposition 1. Given any price distribution \( Q(p) \), the equilibrium probability-of-sale function, \( \varphi(p) \), is clearly a step function; there exist prices \( \hat{p} \) and \( \tilde{p} \) satisfying \( \varphi(p) = 1, \forall p < \hat{p} \) and \( \varphi(p) = \frac{1}{2}, \forall p: \hat{p} < p < \tilde{p} \). At the price \( \hat{p} \) the probability of sale is between \( \frac{1}{2} \) and 1, and at the price \( \tilde{p} \) the probability of sale is between 0 and \( \frac{1}{2} \).

But in equilibrium \( Q(p) \) is the sum of nondecreasing, nonnegative, profit-maximizing price distributions, and \( (p - c)\varphi(p) - k = 0 \), which restricts the equilibrium probabilities at prices \( \hat{p} \) and \( \tilde{p} \). At \( \hat{p} \), \( \varphi(\hat{p}) = \frac{1}{2} \), since otherwise \( \varphi(\hat{p}) < \frac{1}{2} \) and either (i) \( (\hat{p} - c)\varphi(\hat{p}) - k = 0 \) and there exists an \( \epsilon > 0 \) and an interval \([\hat{p} - \epsilon, \tilde{p}]\) on which \( (p - c)\varphi(p) - k > 0 \) or (ii) \( (\hat{p} - c)\varphi(\hat{p}) - k < 0 \) and there exists an \( \epsilon > 0 \) and an interval \([\hat{p} - \epsilon, \tilde{p}]\) on which the distribution has measure zero and \( \varphi(p) < \frac{1}{2}, \forall p \in [\hat{p} - \epsilon, \tilde{p}] \), both of which are contradictions. Similarly, at \( \tilde{p} \) we know that \( \varphi(\tilde{p}) = 1 \), since otherwise there would exist an interval \([\hat{p} - \epsilon, \tilde{p}]\) on which profits were positive or \( \varphi(p) < 1 \).

Since equilibrium profits are strictly increasing on the intervals \([0, \hat{p}]\) and \((\hat{p}, \tilde{p})\), it follows that all of the equilibrium output must be at the prices \( \hat{p} \) and \( \tilde{p} \). And since profits must therefore be zero at these prices, the unique equilibrium prices are \( p_L = \hat{p} = c + k \) and \( p_H = \tilde{p} = c + 2k \). The equilibrium quantities follow immediately.

Q.E.D.

Proof of Proposition 2. If \( F(k) = 1 \), then the competitive equilibrium quantity of high-priced seats is zero. Low-priced seats all sell for \( c + k \), the same as optimal uniform pricing since clearly \( E[w] < 2k \).

If \( F(k) < 1 \) and \( E[w] < 2k \) (case 2), then relative to uniform pricing, equilibrium price dispersion reduces waiting costs by \( E[w|w > k] \) for the \([Q_u - (\overline{Q} - Q_L)(1 - F(k))])\) consumers who would have flown off-peak and now choose to fly peak (equal to the number of high-priced seats not filled by passengers who would have flown peak anyway) and increases by \( E[w|w < k] \) the waiting costs for the \((\overline{Q} - Q_u)F(k))\) consumers who are no longer able to buy a low-priced ticket at the peak time and choose to fly off-peak.

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rather than pay for a high-priced seat. Since capacity costs increase by \( Q_h k \), the net decrease in expected costs is

\[
\left[ Q_n (\overline{Q} - Q_n) (1 - F(k)) E[w | w > k] - (\overline{Q} - Q_n) F(k) E[w | w \leq K] \right] - Q_n k. \tag{A1}
\]

Since \( Q_n / 2 = \overline{Q} - Q_n \) and \( (1 - F(k)) E[w | w > k] - F(k) E[w | w \leq K] = E[w] \), the decrease in expected costs is \( (Q_n / 2) [2E[w | w > k] - 2k - E[w]]. \)

If \( F(k) < 1 \) and \( E[w] \geq 2k \) (case 1), then relative to competitive equilibrium pricing, uniform pricing increases capacity costs by \( (N_1 - Q_n - Q_2) 2k \) and reduces waiting costs by \( E[w | w < k] \) for the \( (Q_n - N_2) \) consumers who fly off-peak but would rather fly peak. Since \( Q_n + 2Q_s = N_1 + N_2 \), it follows that \( N_1 - Q_n - Q_2 = Q_s - N_2 \) so the competitive equilibrium reduces cost by a net \( (Q_s - N_2) [2k - E[w | w < k]] \), which is strictly positive as long as \( Q_s > N_2 \) or \( F(k) > 0 \). Q.E.D.

References


