Long-Lived Consumers, Intertemporal Bundling, and Collusion

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Abstract

In a repeated price game with long but finitely-lived consumers, long-term contracts facilitate collusion. Intertemporal bundling reduces the gains from business stealing but has little effect on the cost of the resulting price war. When consumers anticipate future price wars, the maximum deviation profit is a single period of consumer surplus per consumer. Hence long-term contracts do not increase the incentive to deviate per consumer, but do reduce the the number of consumers currently in the market by locking them into past contracts, so tacit collusion is sustainable for a wider range of discount factors and market structures.

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1 Introduction

Intertemporal bundling is a common business practice. Service contracts for serial products, such as newspapers and magazines, and services, such as cellular telephone and DSL internet access, often require customers to commit to multi-period service agreements.\(^1\) In the cellular telephone industry in particular, analysts have argued that multi-period service agreements are associated with collusion (see Finegold, *Billing World*, 2007). We find theoretical support for the claim that such intertemporal bundling may facilitate collusion. More precisely, we show that if consumers are long-lived, then firms competing with one another will be able to sustain higher profits more easily when they are free to intertemporally bundle their products.

A priori, it is not clear that intertemporal bundling facilitates collusion because increasing the time between firms’ price offers is known to make tacit collusion more difficult. However, intertemporal bundling changes the time between consumers’ purchase decisions, not the time between firms’ pricing decisions. In particular, firms can respond to competitor’s price cuts by starting retaliatory price wars, and in so doing punish the competitor, just as swiftly. This implies that as long as consumers correctly forecast that deviations from the collusive price will be swiftly punished, consumers will accept a deviating firm’s price offer only if it gives them just as much surplus as they could get by waiting for the price war to begin.

To better understand the latter point and more generally to better understand the role of intertemporal bundling in our model, it is useful to ask what would happen if firms were restricted to offer only long-term contracts. Clearly when firms must offer twelve-month contracts that begin on January 1st tacit collusion is more difficult than when firms must offer one-month contracts that begin on the first of each month. This is because intertemporal bundling increases firms’ short-run deviation profits but not their equilibrium path profits.

Now suppose that firms must offer twelve-month contracts but that the contracts can begin on the 1st of any month (not just January 1st). Even if as before every consumer chooses a contract that begins on January 1st, on the equilibrium path the firms’ deviation profits are now much lower. In fact they are the same as when the firms offer one-month contracts. This is because forward-looking consumers anticipate that a price war will begin

\(^1\)Not all multi-period service agreements are examples of intertemporal bundling. In particular, some multi-period service agreements leave the consumer free to change service providers at anytime, and instead only restrict the firm, either by limiting its ability to increase price or to terminate service.
in February, one month after the deviation occurs, so the largest profit a deviator can capture is the total market surplus for just one month. If the deviator attempts to capture more surplus from consumers, the consumers will forgo their consumption for one month and then purchase from rival firms at marginal cost. That is, the deviation profits are proportional to the time between offers, not the length of the contracts.

In fact, intertemporal bundling can make it easier to tacitly collude, not harder. Suppose that each month exactly one twelfth of all consumers are looking to sign a new twelve-month contract. A deviating firm can increase its short-run profits by lowering its price, but it can only capture one month of surplus from one twelfth of the consumers. Yet the cost of a price cut is the forgone future profits from all of its consumers, not just those renewing at the time of the deviation, so tacit collusion is easier to sustain then when firms are restricted to offer only one-month contracts.

Formally, we show that intertemporal bundling facilitates higher profits even when the duration of firms’ contracts is chosen at the time the contracts are offered. We consider a simple model in which consumers live finite lives in overlapping generations. When firms offer lifetime contracts, then in steady state only new consumers are purchasing each period. A deviating firm can only capture one period of surplus from each new consumer, but must give up sales to all future new consumers when it deviates. So as consumers live longer, and firms can intertemporally bundle more, the gains to deviating per consumer are unchanged, but the number of consumers purchasing each period shrinks relative to the number of consumers purchasing in the future, making a deviation less attractive. We also show that holding the consumers’ lifetimes fixed, the number of consumers purchasing each period shrinks relative to the number of consumers purchasing in the future, so tacit collusion becomes easier as contract length increases.

The paper is organized as follows. In Section 2 we provide brief review of the related literature. In Section 3 we describe our model with overlapping generations of finitely-lived consumers and show that intertemporal bundling facilitates tacit collusion. Specifically, we show that in steady state firms can sustain profit levels as high as the monopoly profit with lower discount factors, and in markets with more firms, when they are free to intertemporally bundle than when they are restricted to single-period contracts. Moreover, as consumers live longer, or as the contract length increases holding the consumers’ lifetimes fixed, strictly positive profits become easier to sustain. Section 4 describes the strategies that can be
used to transition from an equilibrium with no intertemporal bundling to the steady-state equilibrium described in Section 3. Section 5 concludes.

2 Literature

Intertemporal bundling is a special case of product bundling. So by analogy, the multi-product bundling literature has suggested several potential rationales for intertemporal bundling, most notably price discrimination and foreclosure.\textsuperscript{2} For example, Adams’s and Yellen’s [1976] results on bundling imply that if consumers’ valuations for a firm’s product or service are negatively correlated across time, a firm with market power can increase their profits (and reduce the deadweight loss) by intertemporal bundling. Similarly, Whinston [1990] implies that intertemporal bundling can be a mechanism for leveraging current market power into the future.

A few papers have looked more specifically at intertemporal bundling, but like the bundling literature more generally, these papers have mainly focused on the role of bundling as a mechanism for price discrimination, not on the role of bundling in facilitating collusion. For example, in the behavioral economics literature, Loewenstein, Donoghue, and Rabin [2003] study a model of projection bias (consumers overestimate the degree to which their future preferences will resemble their current preferences) and conjecture that a firm may use intertemporal bundling to take advantage of consumers with projection bias who currently place a high value on its product. And DeGraba and Mohammed [1999] show that bundling can help a capacity-constrained monopolist extract more surplus from high value consumers. In a two-period model, they show that bundling in period one can induce a buying frenzy among high valuation consumers who anticipate being rationed if they wait until period two when products are sold individually. While the authors describe this tactic as intertemporal mixed bundling, the firm is bundling across units produced in the same period one, not units produced at different times. Mialon and Chen [2008] consider a related two-period model with uncertain demand in period two and show that bundling purchases in period one with an option to purchase in period two outperforms mixed bundling.

Because durable goods are often thought of as generating a flow benefits to the consumer,

\textsuperscript{2}Perhaps the most commonly given justification for intertemporal bundling is that there are fixed costs of establishing service. While these costs undoubtedly exist in many environments, it is less apparent why these costs must be recouped with service agreements rather than installation fees.
our work on intertemporal bundling and collusion is also related to the literature on collusion in durable goods markets. Two related papers in that literature, Ausubel and Deneckere [1987] and Gul [1987], consider oligopoly models of durable-goods pricing and show that the Coase conjecture does not hold in markets with more than one firm because firms find it easier to tacitly collude as the time between offers shrinks.\(^3\) Consumers who rationally anticipate a price war following a price deviation will have an incentive to ignore the price deviation and wait for the price war to begin, so a deviator will only be able to capture the time value of the consumers’ surplus. This is also true in our analysis since we can interpret intertemporal bundling as converting services or perishable goods into durable goods.

Dutta, Matros, and Weibull [2007] consider a related model in which oligopolists sell to overlapping generations of consumers who demand at most one unit of the good in their lifetime. They find that, all else equal, as consumers live longer tacit collusion is more difficult to sustain. The deviation profits increase as the proportion of old consumers grows, because the total number of old consumers in the market who have not yet purchased the good (because their valuations are below the historical market price) is growing relative to the number of new consumers. In contrast, we find that as consumers live longer tacit collusion is easier to sustain. Of course our models are very different because they do not consider intertemporal bundling and we do not consider downward sloping demand.

Finally, our paper is also related to the recent literature on the role of forward contracts in facilitating tacit collusion. In a model of non-durable goods, Liski and Montero [2006] show that tacit collusion is easier to sustain when firms can offer forward contracts. They assume that there are two contracting stages prior to every consumption period. They consider an equilibrium in which consumers sign forward contracts in the first contracting stage. If firms react to a deviation in the first contracting stage by cutting price in the second contracting stage, no firm has an incentive to deviate in the first stage because forward-looking consumers will wait and purchase in the second contracting stage at an even lower price. And since much of the demand is served in the first contracting stage, there is also little incentive to deviate in the second contracting stage.\(^4\) Unlike Liski and Montero, our analysis does not

\(^3\)Bagnoli, Salant, and Swierzbinski [1989] (see also van den Fehr and Kühn [1995]) show that if the set of buyers is finite, a sufficiently patient durable goods monopolist can even achieve the profit of perfect price discrimination.

\(^4\)In a paper that was developed independently, Green and Le Coq [2010] modify Liski and Montero [2006] to allow firms to sell long-term forward contracts at regular, but exogenously given intervals, in addition
depend on the assumption that firm can make multiple price offers prior to each period of consumption.

3 The Model

We generalize a standard infinitely-repeated oligopoly price game by assuming that consumers are long lived and that firms can intertemporally bundle their output. We assume \( n \) infinitely-lived firms sell a homogeneous, non-storable and perfectly divisible product to a continuum of finitely-lived consumers.

We assume there are a continuum of consumers each of which has unit demand, and that their valuations are homogeneous and equal to \( V \) per unit of the good. We normalize the total number of consumers to one. The assumption that consumers are arbitrarily small, combined with the assumption that consumers cannot act collectively, simplifies the formal analysis because we can ignore deviations by consumers (deviations by an single consumer are not payoff relevant for firms).

Consumers live \( l \) periods in overlapping generations, so a fraction \( 1/l \) of the consumers are new to the market each period. Aggregate demand is normalized to one in steady state (Periods \( l \) and beyond) but for simplicity we ignore customers who arrive before Period 1 so aggregate demand is growing until Period \( l \). This assumption implies that the incentives to deviate from the equilibrium strategies are the same in periods 1 to \( l - 1 \) as they are in the steady state periods, so the model characterizes only the steady state incentives. In Section 4 we discuss a more realistic model in which consumers born before Period 1 are in the market in Period 1. This extension allows us to ask how firms can transition to a steady state collusive equilibrium after a number of periods in which they sold short term contracts at competitive prices.

We assume that the firms have zero unit costs, and that firms and consumers have a to selling in a spot market. Like Liski and Montero, they argue that allowing forward contracts allows firms to sustain higher prices, but in contrast to our work, they find that increasing the length of long-term forward contracts makes tacit collusion harder to sustain. Longer contracts reduces firms’ ability to punish one another because firms cannot cut the price of their long-term contracts until the current long-term contracts expire. Firms in our model offer long-term contracts every period to overlapping generations of consumers, and increasing contract length, or equivalently increasing the lifetimes of each generation, makes tacit collusion easier to sustain by reducing the gains to business stealing.
common, strictly positive discount factor, \( \delta \in (0, 1) \).

Each period, firms simultaneously offer contracts, which can vary in both price and length, and then consumers either choose a contract from among the firms’ offers or consume their outside option (that is, give up \( V \)). As is standard in the literature on tacit collusion, we also assume that forward contracts are not feasible (for example, firms cannot offer a contract in Period \( t \) for delivery of the product to consumers arriving in Period \( t + 1 \) or \( t + 2 \)).

Firms’ one-period contract offers are denoted simply by their price, \( p \). Firms’ multi-period contracts are denoted by price-duration pairs, \( \{P,k\} \), where \( P \) is the present discounted value of the stream of per-period payments specified in the contract and \( k \) is the contract length. However, the exposition of the paper is made easier when we suppose that multi-period contracts require a stream of payments as opposed to a single up-front lump-sum payment. In particular this makes it easier to compare a multi-period contract to a series of single-period contracts. Since contracts are binding, and firms and consumers share a common discount factor, any two \( k \)-period contracts whose streams of payments have the same present discounted value are equivalent. So we interpret our contracts as requiring a stream of constant payments. For example, a finite length, multi-period contract, \( \{P,k\} \), can be written as a requirement to pay a stream of \( k \) identical payments, \( p_k \), where \( P = \sum_{i=1}^{k} \delta^{i-1} p_k \).

We assume that consumers are rational, and so we consider equilibria in which consumers correctly anticipate firms’ equilibrium strategies. However, since there are a continuum of consumers, firms’ strategies depend only on other firms’ actions and on the number, or proportion, of consumers choosing each contract, not on the actions of any individual consumer.

We begin by considering the important benchmark case in which intertemporal bundling is not feasible. If the firms are restricted to make one-period offers, the Subgame Perfect Nash Equilibria (SPE) of the game are well-known (see, for example, Tirole, 1988) and is restated in the following claim.

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5Note however that all of the equilibria we analyze are equilibria of a game in which the set of feasible contract offers is more general and includes forward contracts. This is because we can simply assume that firms punish each other for offering forward contracts just as they would punish each other for price cuts. And since consumers are rational, no deviating firm can steal more than one period of consumer surplus from a consumer.

6In our conclusion, we discuss at length how our result is robust to alternative relaxations of the rational consumer assumption.
Claim 1. When intertemporal bundling is not feasible, then i) when \( n \leq \frac{1}{1-\delta} \) any price \( p \in [0, V] \) and any level of profit between zero and the monopoly profit, \( \frac{V}{1-\delta} \), is sustainable in a symmetric SPE, and ii) when \( n > \frac{1}{1-\delta} \), the unique subgame perfect Nash equilibrium outcome is \( p = 0 \) and zero profits (marginal cost pricing).

Note that Claim 1 holds regardless of how long consumers live. That is, the presence of long-lived consumers alone does not facilitate tacit collusion. Also, Claim 1 still holds even though demand is growing for \( l \) periods (the \( n \) and \( \delta \) for which tacit collusion is feasible are characterized by the incentive constraints in the later periods when demand is stationary). However, because we have assumed demand is not stationary, the profits are non-standard. The present value of the monopoly profit, or equivalently the highest possible industry profit (the profit earned when firms set a price \( p = V \)), in the first period is equal

\[
\frac{1}{l} \sum_{i=1}^{l} \delta^{i-1} \frac{V}{1-\delta},
\]

which is lower than the present value of steady state industry profit (in Period \( l \) and beyond),

\[
\frac{V}{1-\delta}.
\]

The difference is that in the first \( l - 1 \) periods the total number of consumers in the market is still growing and is below the steady state level.

In an overlapping generations model with finitely-lived consumers, the most natural way to analyze intertemporal bundling is by assuming consumers purchase lifetime contracts. While we focus primarily on subgame perfect equilibria with lifetime contracts, the broader class of equilibria we consider are those in which the firms offer \( k \)-period contracts on the equilibrium path and offer one-period, marginal cost contracts off the equilibrium path. We also focus only on equilibria in which the contract length, \( k \), and contract price, \( P \), are stationary. We let \( p \) denote the per period price that is equivalent to the contract price \( P \) (i.e., \( P = \sum_{s=1}^{k} \delta^{s-1} p \)).

**Definition.** \( k \)-period Contract Equilibrium: For any \( k \) such that \( l \) is divisible by \( k \), a \( k \)-period contract equilibrium is a subgame perfect equilibrium in which on the equilibrium path firms offer only \( k \)-period contracts \( \left( \sum_{i=1}^{l} \delta^{i-1} p, k \right) \), in which all consumers purchase these contracts in the 1st, \((k+1)\)th, \ldots,\((l-k+1)\)th periods of their life time and divide their purchases equally across the firms, and in which off the equilibrium path, following a deviation by any firm, all firms offer one-period, marginal-cost contracts forever.
When \( k = l \), we refer to a \( k \)-period Contract Equilibrium as a Lifetime Contract Equilibrium.

Although we characterize all of the equilibrium prices that can be sustained in a \( k \)-period Contract Equilibrium, in order to develop the intuition for our result, we begin by focusing on Lifetime Contract Equilibria in which \( p = V \). When consumers live \( l \) periods then an equilibrium exists in which firms offer \( l \)-period contracts with a per period price of \( V \) if their equilibrium continuation profit exceeds their deviation profit.

In each period every firm earns \( \sum_{i=1}^{l} \delta^{i-1} \frac{1}{n} \frac{1}{l} V \) from consumers that are new to the market in that period (the present value of the payments these new consumers make to the firm over their lifetimes), so each firm’s equilibrium continuation profit, i.e., the sum of its profit from new consumers plus the present value of its profit from future consumers, is

\[
\frac{1}{n} \frac{1}{l} \sum_{i=1}^{l} \delta^{i-1} \frac{V}{1 - \delta}.
\]

If a firm deviates from its equilibrium strategy, consumers can purchase from the deviating firm or choose one of two alternatives. The first alternative is to purchase from one of the non-deviating firms at a per period price equal to \( V \) for \( l \) periods and then enjoy a price of zero every period thereafter. However since the contract length is equal to their lifetime, \( l \), this option yields zero surplus (more generally, if \( p < V \) or the contract length, \( k \), is less then consumers’ lifetimes, \( l \), then this option is more attractive). Alternatively, consumers can choose not to purchase this period and then enjoy a price of zero every period thereafter for at total surplus of \( \sum_{i=2}^{l} \delta^{i-1} V/(1 - \delta) \). The latter alternative is clearly more attractive for consumers, so it follows that a deviating firm’s profit is limited to \( V \) per consumer, or a total profit of \( V/l \) since there are only \( 1/l \) new consumers in the market in each period.

So a Lifetime Contract Equilibrium with \( p = V \) is sustainable as long as the equilibrium continuation profit exceeds the maximum deviation continuation profit, or

\[
\frac{1}{n} \frac{1}{l} \sum_{i=1}^{l} \delta^{i-1} \frac{V}{1 - \delta} \geq \frac{V}{l},
\]

or equivalently

(1)

\[
n \leq \sum_{i=1}^{l} \frac{\delta^{i-1}}{1 - \delta}.
\]
The expression on the right hand side is clearly greater than $\frac{1}{1-\delta}$ when $l > 1$, so a per period price of $V$ is easier to sustain with lifetime contracts than in the benchmark model with one-period contracts.

One way to better understand equation (1) is to consider what would happen if we had instead assumed consumers live $l$ periods in non-overlapping generations (a measure of consumers are born in period 1, period $l+1$, and so on). In this case the continuation profit in period 1 (as well as in all periods $il + 1$ for all positive integers $i$) is $\frac{1}{n} \sum_{i=1}^{l} \delta^{i-1} \frac{V}{1-\delta} = \frac{1}{n} \frac{V}{1-\delta}$ and the deviation profit is $V$, so a price $p = V$ is sustainable if and only if $n \leq \frac{1}{1-\delta}$, the standard condition with single period contracts. However, equation (1) is weaker than this condition. In our model both the deviation profit and the continuation profit are lower. The deviation profit is lower because only $\frac{1}{l}$ consumers are available to make purchases in each period. Similarly the continuation profit is lower because each firm sells to only $\frac{1}{n} \frac{1}{l}$ consumers in period $t$. But since these consumers live for $l$ periods as new consumers are arriving, each firm sells to $\frac{1}{n} \frac{2}{l}$ consumers in period $t + 1$, to $\frac{1}{n} \frac{3}{l}$ consumers in period $t + 2$, and sells to $\frac{1}{n}$ consumers in period $t + l - 1$ and beyond. So intertemporal bundling reduces the deviation profits by significantly more than it lowers the continuation profits.

Notice also that the deviation profit earned per consumer is the same regardless of how long consumers live and that lifetime contracts alter the deviation incentives primarily by changing the relative number of consumers buying in the current period versus the future periods. So as consumers’ lifetimes increase, the number of consumers purchasing in the current period falls but the steady state number of consumers served in the future stays the same. The total number of consumers served in the future does fall slightly, but only in the $l - 1$ periods after the current period, not thereafter, so the relative importance of profits from future customers has increased.

Put another way, as consumers live longer lifetimes, the lifetime profit earned per consumer is higher, but deviation profit earned per consumer, which is capped at his or her single-period reservation value, does not increase. If consumers arrive at a constant rate over time and we ignore consumers who arrived in the past (because they are locked into previously signed lifetime contracts), then as lifetimes increase, the deviation profit earned from the current generation of consumers becomes smaller relative to the future profits earned from the both the current and future generations of consumers, and so tacit collusion becomes easier to sustain.
Proposition 1 generalizes this analysis by considering the full set of $k$-period contract equilibria, including equilibria in which the per period price is less than $V$ and equilibria in which the contract length is less than consumers’ lifetimes $l$. However, for simplicity we focus only on contract lengths $k$ where $l$ is divisible by $k$ so that its is feasible to fully serve consumers in every period of their lifetimes using a series of $k$-period contracts.  

**Proposition 1.** Suppose that the length of consumers’ lifetimes, $l$, is divisible by $k$ (i.e., $l/k \in \mathbb{Z}$). A strictly profitable $k$-Period Contract Equilibrium exists if and only if

$$n \leq \sum_{i=1}^{k} \frac{\delta^{i-1}}{1 - \delta}.$$  

The range of per period prices that can be supported in a strictly profitable $k$-Period Contract Equilibrium when one exists is $p \in \left[\frac{n(1-\delta)}{\sum_{i=1}^{k} \delta^{i-1}}, V\right]$ if $n \in \left(\frac{1}{1-\delta}, \frac{\sum_{i=1}^{k} \delta^{i-1}}{(1-\delta)}\right)$ and $p \in [0, V]$ if $n \leq \frac{1}{1-\delta}$, and the associated range of industry profit levels (in Period 1) is

$$\left(0, \frac{\sum_{i=1}^{l} \delta^{i-1}}{l(1-\delta)} V\right) \quad \text{if } n \leq \frac{1}{1-\delta},$$

$$\left[\frac{n \sum_{i=1}^{l} \delta^{i-1}}{l \sum_{i=1}^{k} \delta^{i-1} V}, \frac{1}{l} \sum_{i=1}^{l} \frac{\delta^{i-1}}{1-\delta} V\right] \quad \text{if } n \in \left(\frac{1}{1-\delta}, \frac{\sum_{i=1}^{k} \delta^{i-1}}{(1-\delta)}\right).$$

Proposition 1 characterizes the range of prices and the range of Period 1 profits that can be supported in a $k$-Period Contract Equilibrium. The range of Period 1 profits are also equal to the range of continuation payoffs that the industry can earn in any period of the game. However recall that industry profit in steady state (periods $l$ and beyond) include revenues from contracts signed in the past, so industry steady state profits are $\frac{p}{1-\delta}$. This is true because we are interpreting long-term contracts as stipulating that a payment $p$ be made in each period in which the good are service is delivered. So clearly in steady state firms extract all of the consumer surplus when $p = V$. However in Period 1 the industry profit level is necessarily lower than $\frac{p}{1-\delta}$ because there are no payments from contracts signed prior to Period 1. Industry profits do not reach the steady state level until the number of consumers reaches the steady state level in Period $l$.  

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Note that the proposition applies to any value of $l$, even a prime number. However when $l$ is prime, the proposition holds only for $k = 1$ and $k = l$.  

11
Comparing Claim 1 and Proposition 1 it is clear that intertemporal bundling using lifetime contracts helps the industry sustain tacit collusion when the number of firms is between \( \frac{1}{1-\delta} \) and \( \sum_{j=1}^{l} \frac{\delta^{j-1}}{1-\delta} \). Two examples illustrate the power of intertemporal bundling. First, suppose that \( \delta = 0.9 \) and \( l = 12 \). In this case, when intertemporal bundling is not feasible, tacit collusion can be sustained in an industry with up to ten firms. But when intertemporal bundling is feasible, tacit collusion can be sustained with up to 71 firms (\( \sum_{j=1}^{12} \frac{(0.9)^{j-1}}{1-(0.9)} \approx 71.76 \)).

Second, consider a common duopoly setting, i.e., \( n = 2 \), and suppose that \( l = 12 \). Then without intertemporal bundling tacit collusion is feasible if \( \delta \geq \frac{1}{2} \), but with intertemporal bundling tacit collusion is feasible if \( \delta \geq \hat{\delta} \approx 0.293 \). That is, when the discount factor lies between .293 and .5, tacit collusion is feasible when firms use intertemporal bundling (in a Lifetime Contract Equilibrium) but not otherwise.

Proposition 1 clearly implies that increasing \( l \) and \( k \) facilitates collusion.

**Corollary 1.** For any given \( l \), the number of firms among which a \( k \)-Period Contract Equilibrium is sustainable increases in \( k \). Also, the number of firms among which a Lifetime Contract Equilibrium is sustainable increases in the length of lifetime \( l \).

Corollary 1 follows immediately the fact that \( \sum_{i=1}^{k} \frac{\delta^{i-1}}{1-\delta} \) is increasing in \( k \) and the fact that \( \sum_{j=1}^{l} \frac{\delta^{j-1}}{1-\delta} \) is increasing in \( l \).

Figure 1 shows the steady state industry profits that can be sustained in a \( k \)-Period Contract Equilibrium and a Lifetime Contract Equilibrium. The region \( A_0 \) depicts the set of feasible steady state equilibrium profits when \( n \leq \frac{1}{1-\delta} \) whether or not intertemporal bundling is feasible. That is, region \( A_0 \) depicts the steady state profits under Claim 1. The region \( A_0 \cup A_1 \) depicts the set of steady state equilibrium profits that are sustainable, as a function of the number of firms, when firms can intertemporally bundle and choose to play a \( k \)-Period Contract Equilibrium and consumers live \( l \) periods, where \( k < l \). The region \( A_0 \cup A_1 \cup A_2 \) denotes the set of equilibrium profits that are sustainable when firms play Lifetime Contract Equilibrium and consumers live \( l \) periods.\(^9\)

\(^8\)Note that \( \hat{\delta} \) is the solution to \( 2 = \sum_{j=1}^{12} \frac{\delta^{j-1}}{1-\delta} = \frac{1-\delta^{12}}{(1-\delta)^{12}} \).

\(^9\)Note that the number of firms, \( n \), is an integer, but for ease of illustration Figure 1 treats \( n \) as any positive real number.
Figure 1: The Impact of Consumers’ Lifetimes on Feasible Profits in Lifetime Contract Equilibria: \( k < l \) (\( A_0 \cup A_1 \)) versus \( k = l \) (\( A_0 \cup A_1 \cup A_2 \))

4 Equilibrium Transition to Collusion in Steady State

In the previous section, we characterized the impact of intertemporal bundling on steady state industry profits, but did not describe how colluding firms could transition to the steady state equilibrium. This was done by that there were no consumers in the market born before Period 1.

In this section we try to consider a more realistic environment that explains how tacit collusion could arise when the market first forms on when collusion had not occurred in the past. In particular, suppose that historically firms offered only single-period contracts and priced at marginal cost (or equivalently didn’t exist) and that firms are only considering colluding (or are only able to collude) beginning in Period 1. How would firms transition to the steady state equilibrium?

More formally, let’s call the model analyzed above the steady state game and now consider
a new, more realistic game in which in Period 1 there is a unit mass of consumers and that a fraction $1/l$ of these consumers have one period of their lives remaining, another fraction $1/l$ of these consumers have two periods of their lives remaining, and so on. So in Period 2 and beyond, a fraction $1/l$ of consumers leave the market and are replaced by a fraction $1/l$ consumers who live for $l$ periods.

We can easily see that when strictly positive profits are feasible in the steady state game then strictly positive profits must also be feasible in the new game. Consider any equilibrium of the steady state game in which firms earn strictly positive profits. Then an equilibrium of the new game can be constructed in which firms sell $l$-period contracts at marginal cost until Period $l$ and thereafter play the equilibrium of the steady state game. In Periods 1 to $l - 1$ all of the new consumers will purchase $l$-period contracts, so regardless of what the other consumers choose to do, in Period $l$ all consumers born before period $l$ are locked into $l$-period contracts from prior periods, so the game beginning at Period $t$ is identical to the steady state game analyzed above. Because firms earn zero profits before period $l$ and any deviation can be punished by zero profits forever, it follows that this is a SPE and that any SPE of the steady state game is a SPE of the continuation game beginning in Period $t$.

Note that in this equilibrium first period profits in the new game are lower than in the most profitable SPE of the steady state game. Even though demand in the new game is at the steady state level in period 1, it isn’t necessarily the case that firms can earn the steady state profits. In fact, when $n > \frac{1}{1-\delta}$ we now show that firm profits are bounded strictly below $\frac{V}{1-\delta}$, the highest sustainable profit in the steady state game.

**Proposition 2.** For $n \leq \frac{1}{1-\delta}$ the industry profit that can be supported in any symmetric SPE is at most $\frac{V}{1-\delta}$. For $n > \frac{1}{1-\delta}$, the industry profit that can be supported in any symmetric SPE is at most $\frac{n}{n-1} \delta \frac{V}{1-\delta}$.

Intuitively, in order for industry profits to reach $\frac{V}{1-\delta}$ firms must extract all of the consumer surplus. But if firms extract all of the surplus, then a firm could deviate by offering a one period contract at price $V$ and capture the whole market. And when $n > \frac{1}{1-\delta}$, it follows that $\frac{1}{n} \frac{V}{1-\delta} < V$ so this deviation must be profitable. So profits must be strictly lower than $\frac{V}{1-\delta}$.

We now show that the upper bound in Proposition 2 is a tight upper bound, that is a SPE exists that attains the upper bound.

**Proposition 3.** For $n \leq \frac{\sum_{i=1}^{k} \delta^{i-1}}{(1-\delta)}$, the upper bound, $\frac{n}{n-1} \delta \frac{V}{1-\delta}$, characterized in Proposition 2 is attainable in a SPE with intertemporal bundling.
Intuitively, the most profitable time for firms to deviate is in Period 1 when all consumers are in the market, so whenever intertemporal bundling is necessary to sustain the monopoly profits in the steady state (i.e., when \( n > \frac{1}{1-\delta} \)), the firms need to offer consumers some surplus up front in order to limit the number of consumers who are in the market in subsequent periods. We show that the best firms can do is give enough surplus to consumers in Period 1 such that they are willing to sign contracts that enable the firm to shift the demand to the steady state demand in Period 2 and beyond. Firms do so by offering a menu of contracts such that each customer signs a contract in Period 1 which lasts the remainder of his or her lifetime.

Propositions 2 and 3 highlight that intertemporal bundling increases profits because it induces consumers to renew their contracts at different times. Put another way, if consumers were finitely lived but did not live in overlapping generations, then intertemporal bundling (particularly with lifetime contracts) would not make collusion easier.

5 Conclusion

In this paper, we demonstrated that intertemporal bundling can help soften competition by facilitating tacit collusion. Tacit collusion is easier to sustain when firms offer intertemporally bundled contracts because a deviating firm can steal business only in one market segment at one point of time but it can be punished in all of the market segments.

Our results relied on several strong assumptions. First, we assumed that consumers are small and unable to act collectively. If a single consumer represented a large portion of a firm’s business, or if a large measure of consumers could act collectively, firms’ ability to tacitly collude would be diminished.

More importantly, we assumed consumers were rational and exploited the fact that rational consumers are forward looking and anticipate a price war whenever they observe a deviation by any firm. Although this assumption is widely adopted in oligopoly pricing models, from some readers’ perspective, our results may seem to rely very strongly on this assumption. In particular, consumers are hurt by their own rationality.

Another strong assumption was that all consumers lived the same finite lengths. However relaxing this assumption would be straightforward, particularly if we allowed firms to offer long-term contracts as well as single-period contracts. For example, if consumers lived a
mixture of 3 and 4 periods, the firms could offer 3-period contracts that consumers would sign when they first arrived in the market and one period contracts that the longer-lived consumers would sign after their long-term contract ended. The presence of consumers signing short term contracts would make tacit collusion more difficult, but in steady state intertemporal bundling would still make tacit collusion easier.

One way to check the robustness of our findings is to assume some consumers are naïve and always expect future prices to be the equilibrium prices, even after seeing deviations. Of course, this means that firms can easily exploit consumers, and as a consequence tacit collusion is harder to sustain – a deviating firm can capture all of the future profit from the naïve consumers currently in the market by offering them lifetime contracts. Nevertheless, in our work in progress, we show (with a very similar model) that when some (and possibly all) consumers are naïve, the most profitable equilibrium in which firms offer lifetime subscription contracts on the equilibrium path is more profitable than the most profitable equilibrium in which firms offer one-period contracts on the equilibrium path. In the latter case, tacit collusion is particularly difficult because prices must be low enough to prevent business stealing by a firm that can offer long term contracts to a large number of consumers. However, in the former case, because some consumers’ current and future demands are already under contract, a deviation is less profitable when firms offer long-term contracts because there is less business to steal whether the deviating firm offers one-period or long-term contracts and whether consumers are naïve or not. In other words, intertemporal bundling can facilitate collusion even when some consumers are naïve.

Our results are also consistent with a simpler model of consumer behavior. For example, if we assume consumers use a rule of thumb which is to wait and see what happens whenever they observe a deviation from equilibrium behavior, then the equilibrium firm behavior would be almost identical. Of course, it might seem arbitrary to assume they wait just one period, and the costs of waiting one period can be high if firms actually revert to equilibrium pricing. But this suggests that a wide range of models with behavioral consumers would give similar predictions.

We also assumed consumers were homogenous. In an earlier version of the paper we analyzed heterogeneous valuations. Heterogeneity in the valuations implies that deviations are more profitable (at least in the case that the only profitable deviations are prices significantly below the equilibrium price) so the range of discount factors for which tacit collusion
is feasible with intertemporal bundling is diminished, but otherwise has little implication for the ability of bundling to facilitate tacit collusion.

Finally, our analysis relied on the implicit assumption that a firm’s price cut is observed by its competitors even when consumers do not purchase at the lower price. This assumption guarantees that a deviating firm cannot steal more than one period of the surplus from all the available consumers. If it tries to steal more, all consumers will wait for the ensuing price war. However, we believe our results hold when price cuts are observed only when resulting sales are positive. Consumers will accept any offer below the equilibrium path price if they anticipate that no one else will accept it, and they will reject any offer if they think other consumers will accept it and waiting for the price war to begin is more attractive. So a symmetric mixed strategy equilibrium should exist, at least in a model with finite consumers. In equilibrium, following a deviation consumers mix between accepting and rejecting, and so price deviations will attract some business. Of course, if consumers are arbitrarily small, price deviations will attract an arbitrarily small portion of the market. Alternatively, it may be possible to construct an asymmetric pure strategy equilibrium in which exactly one consumer accepts the offer and the rest wait for the price war to begin. Since the price war will not begin otherwise, the designated consumer will always accept a lower price. In either case, intertemporal bundling will make supporting positive profits easier because it limits deviating firms ability to capture all of the available consumer surplus for more than a single period.

We think the Lifetime Contract Equilibria are realistic and empirically relevant. In these equilibria firms write overlapping subscription contracts with their consumers similar to the contracts used in the cellular telephone industry. Consumers typically sign annual or two-year contracts (when they get a new phone or upgrade their existing one) and these contracts are signed in different months of a year.

Our paper has some important empirical implications. First, we predict that firms’ margins may be sensitive to the expected lifetime of consumers and to the feasibility of long-term contracts. If consumers are not long-lived, then firms will be unable to soften competition using long-term contracts. Similarly, we predict firms’ margins may be sensitive to the ability of consumers to forecast their demands over time. Absent predicable demand, long-term contracts would be inefficient and unable to facilitate tacit collusion.

While we show intertemporal bundling facilitates tacit collusion, it is important to add
that there are also many efficiency explanations for intertemporal bundling. Subscription contracts reduce transactions costs, facilitate firm production planning (as do other advance purchase contracts), and speed delivery time for consumers.
Appendix

Proof of Proposition 1:

First, consider each firm’s equilibrium profit in any arbitrary period $t \geq l$ (any period after demand has reached the stationary level) and suppose that every consumer that arrived before period $t$ signed $k$-period contracts when they arrived in the market and signed new $k$-period contracts each time the previous $k$-period contract expired. Then in Period $t$, the consumers who are willing to sign a new contract are the new consumers who arrive in Period $t$ and consumers whose contracts have just expired – that is, the customers who arrived in periods $t - k, t - 2k, \ldots, \text{and } t - (l/k - 1)k$. The other consumers who arrived before Period $t$ are already locked into an existing contract and since their payment is sunk, we can think of them as already purchasing the good at a zero price.

So the total number of the available consumers in Period $t$ is $\frac{l}{k}$. If the firm offers the equilibrium contract, then in Period $t$ the firm will earn a profit of $\sum_{i=1}^{k} \delta^{i-1} p$ from each of its consumers, or a total of $\frac{1}{n} \frac{l}{k} \sum_{i=1}^{k} \delta^{i-1}$ since there are a total of $\frac{l}{k}$ consumers and the firm splits these with a total of $n$ firms. If the firm continues to offer the equilibrium contract in the future then the present value of its equilibrium continuation profit, excluding profits from previously signed contracts, is equal to

$$\pi^E = \frac{1}{n} \frac{p}{k} \frac{l}{k} \frac{1}{1 - \delta}.$$

Next consider the firm’s continuation profit if it deviates. When a firm deviates, consumers can either accept the deviating firm’s offer, accept a non-deviating firm’s $k$-period contract offer at a price $p \sum_{i=1}^{k} \delta^{i-1}$, or abstain from consuming for one period and then pay a price of zero for the remainder of his or her lifetime. Abstaining for one period allows the consumer to consume every period for the rest of his or her lifetime. Abstaining for one period allows the consumer to consume every period for the rest of his or her lifetime at a one-time cost of $V$, the opportunity cost of forgoing his or her valuation $V$ for one period, and purchasing from a non-deviating firm allows the consumer to consume for the rest of his or her life at cost of $p \sum_{i=1}^{k} \delta^{i-1}$. So accepting a non-deviating firm’s $k$-period contract is strictly more attractive than waiting one period if and only if $p \sum_{i=1}^{k} \delta^{i-1} < V$. Since the deviating offer has to be (weakly) more attractive than the consumer’s outside option, it follows that the
highest profit a deviating firm can earn is

\[ \pi^D = \frac{1}{k} \min \left\{ p \sum_{i=1}^{k} \delta^{i-1}, V \right\}. \]

Collusion is sustainable in Period \( l \) and beyond if and only if \( \pi^E \geq \pi^D \), or equivalently \( k\pi^E \geq k\pi^D \), or

\[ (2) \quad \frac{1}{n} \frac{p \sum_{i=1}^{k} \delta^{i-1}}{(1 - \delta)} \geq \min \left\{ p \sum_{i=1}^{k} \delta^{i-1}, V \right\}. \]

Next, notice that in any period \( s < l \) there are weakly fewer consumers in the market than in Period \( l \) or beyond (the number of consumers is \( \frac{1}{k} \), the same as Period \( l \) and beyond, when \( k = l \) and is less than or equal to \( \frac{1}{k} \) when \( k < l \)). Let \( m_s \leq \frac{1}{k} \) denote the number of consumers without prior contracts in period \( s \), let \( \pi^D_s \) denote the deviation profit in period \( s \), and let \( \pi^E_s \) denote the equilibrium continuation profit in period \( s \). Clearly \( \pi^D_s = \frac{m_s}{1/k} \pi^D \) and \( \pi^E_s \geq \frac{m_s}{1/k} \pi^E \), because the number of available consumers per period is going to grow from \( m_s \) to \( \frac{1}{k} \), so it follows that \( \pi^E_s \geq \pi^D_s \), \( \forall s \), i.e. collusion is sustainable in every period \( s < l \), if and only if \( \pi^E \geq \pi^D \), i.e. if and only if (2) holds.

Having established that (2) is necessary and sufficient for collusion, we now consider what prices, \( p \), can be sustained. First, suppose \( n \leq \frac{1}{1-\delta} \). Then clearly

\[ \frac{1}{n} \frac{p \sum_{i=1}^{k} \delta^{i-1}}{(1 - \delta)} \geq p \sum_{i=1}^{k} \delta^{i-1}, \]

and since

\[ p \sum_{i=1}^{k} \delta^{i-1} \geq \min \left\{ p \sum_{i=1}^{k} \delta^{i-1}, V \right\}, \]

it follows that (2) is satisfied for all \( p \in (0, V] \).

Next, suppose \( n > \frac{1}{1-\delta} \). In this case

\[ \frac{1}{n} \frac{p \sum_{i=1}^{k} \delta^{i-1}}{(1 - \delta)} < p \sum_{i=1}^{k} \delta^{i-1}, \]

so (2) is satisfied only if

\[ (3) \quad \frac{1}{n} \frac{p \sum_{i=1}^{k} \delta^{i-1}}{(1 - \delta)} \geq V. \]
Since \( V \geq \min \left\{ p \sum_{i=1}^{k} \delta_{i}, V \right\} \), it follows that (3) is also a sufficient condition for (2) to hold. Since (3) is equivalent to
\[
p \geq \frac{n (1 - \delta)}{\sum_{i=1}^{k} \delta_{i} - 1} V
\]
and since \( p \leq V \), it follows that (2) is satisfied if and only if
\[
(4) \quad p \in \left[ \frac{n (1 - \delta)}{\sum_{i=1}^{k} \delta_{i} - 1} V, V \right].
\]

In summary, when \( n \leq \frac{1}{1 - \delta} \), a strictly profitable \( k \)-Period Contract Equilibrium with any \( p \in (0, V] \) is sustainable, and when \( n > \frac{1}{1 - \delta} \), a strictly profitable \( k \)-Period Contract Equilibrium is sustainable when the price is sufficiently high – in the range given by (4).

Because new consumers are arriving at a rate \( \frac{1}{l} \) per period for the first \( l \) periods, the period one equilibrium industry profit for a given \( p \) is
\[
p \sum_{i=1}^{l} \delta_{i} - 1, \quad \frac{l (1 - \delta)}{1}
\]
so when \( n \leq \frac{1}{1 - \delta} \) the sustainable first period equilibrium industry profit is in the range \( (0, \frac{n (1 - \delta)}{\sum_{i=1}^{l} \delta_{i} - 1} V) \). When \( n \in \left( \frac{1}{1 - \delta}, \frac{n (1 - \delta)}{\sum_{i=1}^{l} \delta_{i} - 1} \right) \), a strictly profitable \( k \)-Period Contract Equilibrium with any \( p \in \left[ \frac{n (1 - \delta)}{\sum_{i=1}^{k} \delta_{i} - 1} V, V \right] \) is sustainable, so the sustainable first period equilibrium industry profit is in the range \( \left[ \frac{n \sum_{i=1}^{l} \delta_{i} - 1}{l (1 - \delta)} V, \frac{\sum_{i=1}^{k} \delta_{i} - 1}{l (1 - \delta)} V \right] \). And finally, when \( n > \frac{\sum_{i=1}^{k} \delta_{i} - 1}{l (1 - \delta)} \), the price range in (4) is the empty set, so no strictly profitable \( k \)-Period Contract Equilibrium exists.

**Proof of Proposition 2:**

If \( n \leq \frac{1}{1 - \delta} \), clearly the highest sustainable profit is \( \frac{V}{1 - \delta} \), since this can be supported with firms offering single-period contracts on the equilibrium path and static Nash reversion and since the firms are capturing the entire surplus.

Now suppose instead that \( n \geq \frac{1}{1 - \delta} \). Consider any SPE. Let \( \pi \) denote each firm’s equilibrium profit, and let \( \pi^D \) denote the maximum first-period deviation profit for any individual firm. Let \( F_1 \) denote the set of contracts offered to consumers by each firm in Period 1. Let \( f \) be an element of \( F_1 \) and \( P_f \) the associated equilibrium price (for multiple-period contracts \( P_f \) is the net present value of the payment each period).

Suppose \( \min_{f \in F_1} P_f \geq V \). This implies that conditional on observing a deviation, consumers’ best outside option is to abstain from consumption for one period and then purchase
at a price of 0 every period afterwards. So \( \pi^D = V \) (that is, in Period 1 a firm can capture the entire market by offering a single-period contract at a price of \( V \)). However in equilibrium it must be the case that \( \pi \geq \pi^D \), so \( \pi \geq \pi^D = V \), and since \( n > 1/(1 - \delta) \), this implies the industry profit satisfies

\[
n\pi \geq nV > \frac{V}{1 - \delta},
\]
which is impossible since industry profit cannot exceed total surplus. So it follows that in any SPE equilibrium \( \min_{f \in F} P_f < V \).

Now suppose instead that \( \min_{f \in F_1} P_f < V \). It follows that conditional on observing a deviation consumers’ best outside option is to purchase the least expensive contract available for \( \min_{f \in F} P_f \) and then purchase at a price of 0 every period afterwards. So

\[
(5) \quad \pi^D = \min_{f \in F} P_f,
\]
that is, in Period 1 the highest profit that a deviating firm can earn is the price that consumers pay for their outside option.

Note that equilibrium consumer surplus must be at least equal to \( V - \min_{f \in F_1} P_f \) since this is the surplus consumers get from purchasing the lowest priced available contract and consuming only in Period 1, so

\[
(6) \quad \frac{V}{1 - \delta} - n\pi \geq V - \min_{f \in F_1} P_f,
\]
or

\[
(7) \quad n\pi \leq \min_{f \in F_1} P_f + \delta \frac{V}{1 - \delta}
\]

In any equilibrium, \( \pi^D \leq \pi \), so (5) and (7) imply ;

\[
(8) \quad \min_{f \in F_1} P_f \leq \frac{1}{n} \left( \min_{f \in F_1} P_f + \delta \frac{V}{1 - \delta} \right).
\]
or

\[
(9) \quad \min_{f \in F_1} P_f \leq \frac{\delta}{n - 11 - \delta} \frac{V}{1 - \delta}.
\]

Inequalities (7) and (9) together imply that every SPE must satisfy

\[
n\pi \leq \frac{\delta}{n - 11 - \delta} \frac{V}{1 - \delta} + \delta \frac{V}{1 - \delta}
\]
or

\[
n\pi \leq \frac{n}{n - 11 - \delta} \frac{\delta V}{1 - \delta}.
\]
Proof of Proposition 3:

Suppose the firms offer a menu of contracts in Period 1 consisting of 1-period, 2-period, \ldots, \(l\)-period contracts and each specifies a first-period payment of \(p_1\) and a payment of \(V\) every period thereafter. Suppose every contract offered in Period 2 and beyond specifies a payment \(V\) every period. And suppose that following any deviation by a any firm, all firms revert to single-period marginal cost contracts forever. If these strategies are subgame perfect equilibrium strategies, then industry profits will be \(p_1 + \frac{\delta V}{1-\delta} \).

By Proposition 1, these strategies are a subgame perfect equilibrium from Period 2 onwards as long as
\[
n \leq \sum_{i=1}^{l} \frac{\delta^{i-1}}{(1-\delta)}.
\]
Furthermore, these strategies are an equilibrium in Period 1 as long as
\[
\frac{1}{n} \left( p_1 + \frac{\delta}{1-\delta} V \right) \geq p_1,
\]
since \(p_1\) is the maximum deviation profit in Period 1 (consumers’ best outside option is to accept a competitors’ single-period offer of \(p_1\) and pay 0 every period thereafter). If \(n \leq \frac{\delta}{1-\delta}\) then clearly this is satisfied for \(p_1 = V\) which is the highest price consumers are willing to pay. If \(n > \frac{\delta}{1-\delta}\) then the largest value of \(p_1\) that can be supported in equilibrium
\[
p_1 = \frac{1}{n-1} \frac{\delta}{1-\delta} V,
\]
and the equilibrium industry profit is
\[
p_1 + \frac{\delta}{1-\delta} V = \frac{n}{n-1} \frac{\delta}{1-\delta} V,
\]
which is the upper bound specified in Proposition 2. \(\square\)
References


