Buyer groups as strategic commitments

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Buyer cooperatives, buyer alliances, and horizontal mergers are often perceived as attempts to increase buyer power. In contrast to prior research emphasizing group size, I show that even small buyer groups composed of buyers with heterogeneous preferences can increase price competition among rival sellers by committing to purchase exclusively from one seller. Without transfer payments, at least one buyer group exists for each pair of sellers and buyer groups membership is chosen to achieve indifference between the two sellers. With transfer payments, and just two sellers, the grand coalition is a coalition-proof subgame perfect equilibrium (CP-SPNE), though equilibria with arbitrarily many buyer groups also exist. With three sellers (and with more sellers when the distribution of buyers is symmetric), a CP-SPNE always exists, all coalition-proof equilibria are payoff equivalent and have at least one buyer group for each pair of firms, so the grand coalition is not an equilibrium.

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1. Introduction

Firms often form purchasing alliances, or merge, in order to obtain lower prices from their suppliers. Often these are formal, long-term relationships. In the US, independent retail grocers have long used co-operatives for this purpose, including the Independent Grocer’s Association (or IGA), a well-known grocery chain. IGA forms formal, negotiated relationships with large multiproduct food manufacturers, who are then designated part of The Red Oval Family, and requires that all IGA member grocery stores carry products from a Red Oval Family manufacturers.2 Also in the US, cable television franchises use buyer groups to negotiate with content providers, and prescription benefit managers, such as Medco, negotiate with pharmaceutical companies on behalf of the members of Health Maintenance Organizations (HMOs) and Preferred Provider Organizations (PPOs). These buyers obtain lower prices by limiting the number of prescription drugs on the HMO’s or

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2 The Red Oval Family is described on IGA’s web page: http://www.iga.com/. Similarly, Ace Hardware, TruServ, and Do It Best are three large US co-operatives who negotiate with manufacturers on behalf of their member retail hardware stores. While their web sites do not indicate that they require member stores to purchase products from the manufacturers they negotiate with, Do It Best’s product line is bundled with merchandising and marketing services in a way that appears to limit the attractiveness of individual product deviations by its members. Similarly, Ace Hardware appears to bundle distribution and other services with its products, including an unusually large line of private label products (i.e., Ace brand), which also limits product level deviations by members.
PPO’s drug formularies (preferred prescription drug lists). In Europe, horizontal mergers between supermarket and convenience store chains have been closely scrutinized by antitrust authorities as possible attempts to increase buyer power. In some cases store purchase alliances are even formed for a single purchase; in 2003, four airlines, Air Canada, Luftansa, SAS, and Australian Airlines, all members of the Star Alliance (a code-sharing airline network), joined together for the purpose of purchasing aircraft.

This paper argues that buyer groups can make buyers better off by increasing the degree of rivalry among the suppliers of horizontally differentiated products. When purchasing independently, buyers who exhibit a preference for a particular product are at a disadvantage. When suppliers can price discriminate, the idiosyncratic difference between the value a buyer places on his or her preferred product and the value of his or her second choice can be extracted by the seller through a higher price. Groups of heterogeneous buyers can mitigate this disadvantage by forming a buyer group and committing to purchase exclusively from just one supplier. Although some buyers end up consuming their less preferred product, the resulting loss in buyer surplus is more than offset by the gain in surplus that results from lower prices. In contrast to the prior literature on buyer groups, the motivation for buyer groups in this paper does not stem from the size of the groups, but rather from their composition and their ability to commit to purchase exclusively from one seller.

Consider the following simple numerical example. Suppose that Firm $A$ and Firm $B$ each have constant unit costs of $1$. Suppose that Buyer 1 values one unit of Firm $A$’s output at $10$ and one unit of Firm $B$’s output at $9$ (and derives no value from additional units of either firm’s output). Conversely, suppose that Buyer 2 values one unit of Firm $A$’s output at $9$ and one unit of Firm $B$’s output at $10$. In the absence of a buyer agreement, suppose that the firms simultaneously make separate take-it-or-leave-it price offers to each of the buyers and the buyers subsequently choose the product that maximizes their surplus. In equilibrium, Firm $A$ offers to sell one unit at a price of $2$ to Buyer 1 and a second unit to Buyer 2 for $1$, and Firm $B$ offers to sell to Buyer 2 for $2$ and to Buyer 1 for $1$. In response to these offers Buyer 1 buys from Firm $A$ and Buyer 2 buys from Firm $B$. Each firm’s profit is $1$, and each buyer’s surplus is $8$.

The two buyers can increase their payoffs by forming a buyer group and committing to purchase their output exclusively from either Firm $A$ or Firm $B$. The two buyers collectively value two units from Firm $A$ at $19$ and two units from Firm $B$ at $19$. So in equilibrium the prices are $2$ for two units of output from each firm, and the buyers mix between buying from Firm $A$ and Firm $B$. Note that the increased competition leads to lower unit prices. Each firm’s equilibrium profit is $0$, and each buyer’s expected surplus is $8.50$. The two buyers are strictly better off than before, jointly receiving $1$ in additional consumer surplus. However total surplus has fallen because both consumers purchase from the same firm even though it is socially efficient from them to purchase from different firms.

The paper analyzes a more general version of this example with a continuum of heterogeneous consumers, with an arbitrary finite number of firms, and with (or without) transfer payments. I also focus on buyer groups that are coalition proof – no set of buyers who could do strictly better forming a new group. In the absence of transfer payments, groups increase buyer surplus because the group’s membership is chosen in order to make the group indifferent between two sellers and because this induces the two sellers to make price offers at marginal cost, as in the example above. When transfer payments are feasible, groups can form that are not indifferent. However in this case, the gains are not necessarily distributed equally across heterogeneous buyers and some buyers may not benefit from joining buyer groups at all. Keep in mind though that buyer groups are often formed for the purchase of multiple items, in which case it is easy to think of buyers as benefitting from the joining the buyer group for some purchases but not others.

Much previous work on buyer power has focused on the relationship between a single buyer and a single seller. What determines the rents, or surplus, that the buyer will receive? First, buyer power depends on the timing of the parties offers. For example, the buyer’s surplus rises when a monopoly seller loses the ability to commit to a take-it-or-leave-it offer. Second, the buyer’s surplus may increase when buyers have private information (see, for example, Maskin and Riley, 1984). Conditional on trade, privately informed individuals generally capture information rents. Third, buyer power increases when buyers have stronger outside options.

In a single seller, multiple buyer environment, buyers may be able to increase their negotiating power by forming groups. While individuals can only capture a share of the marginal surplus they create, groups can capture a share of the inframarginal surplus created by their members. This inframarginal surplus is higher when the seller has decreasing returns to scale in production, when the group is larger, and when there are positive consumption externalities (see Horn and Wolinsky, 1988a, 1988b; Stole and Zweibel, 1996; Chipty and Snyder, 1999; Chae and Heidhues, 2004; Inderst and Wey, 2003, Wall Street Journal, May 20, 2003).

3 Consumers who join PPOs benefit from lower prices for these medications, but their choice of drugs is restricted. As importantly, the doctors’ are restricted by these formularies when they choose which drugs to prescribe (except when alternatives are medically necessary).

4 The Office of Fair Trade in the UK expressed these concerns in a recent statement to the London Stock Exchange (see AFX News, 9 March 2006).

5 While these four airlines own airplanes previously purchased from a variety of manufacturer’s, in 2003 they agreed to buy over 100 planes in a single model aircraft, either Airbus’ A318, Boeing’s 777, Embraer’s 170, Bombardier’s CRJ700, or Bombardier’s CR900. It is estimated that the total order will be as much as $4.5 billion (see “Airlines Move Toward Buying Their Planes Jointly in Alliances,” Wall Street Journal, May 20, 2003).

6 While this paper takes the ability of buyer groups to commit as exogenous, the examples suggest that the commitment device may be facilitated by the fact that the buyer group purchases many products and that group members outside option, to leave the group and buy individually, cannot be exercised on each individual product, but only the bundle.

7 If the firms’ offers can be arbitrary non-linear schedules of their own sales, or more generally, the sales of both firms, then this game has other Nash equilibria as well. However the equilibrium described here is the only one that satisfies the truthful Nash equilibrium refinement proposed by Bernheim and Whinston (1986) and is the one that yields the highest profit for the sellers (see O’Brien and Shaffer, 1997).
This is also true when the seller has capacity constraints and buyers have heterogeneous valuations. The seller's outside option when negotiating with an individual is the willingness to pay of the highest valuation excluded buyer, however when negotiating with a buyer group of size $x$, the outside option is the willingness to pay of the next $x$ individuals, which by construction must be lower (see Inderst and Wey, 2007). Atakan (February 2008) suggests that some of these results may not be robust to reasonable changes in the timing of the bargaining game. The benefits of collective bargaining by buyers may be even greater when buyers have private information. Che (2002) considers essentially this problem in an analysis of the impact of plaintiff groups (i.e., class action lawsuits) on settlement negotiations with a defendant.

Only a handful of papers have considered buyer groups in a multiple seller setting. Two papers have considered the role of exclusive purchase agreements in a multiple seller setting. O’Brien and Shaffer (1997) consider exclusive purchase agreements in a model with a single buyer. They show that a monopsony retailer is better off when it can commit to purchase exclusively from either of two suppliers. As in this paper, the exclusive purchase commitment intensifies supplier price competition, but is ex post inefficient. However, my paper is more general because it analyzes the conditions under which buyer groups are able to form to exploit exclusive purchase agreements, it shows more precisely that it is a buyer group’s composition, not its size or bargaining power, that determines the effectiveness of exclusive purchase agreements, and it considers more than two sellers.

Inderst and Shaffer (2007), which was developed independently, considers exclusive purchase commitments in a model of retail mergers. In their paper, two differentiated retailers purchase from two differentiated suppliers. They find that retailers may obtain better prices if they merge than if the purchase independently. Their result is similar because they assume an exclusive purchase commitment is only feasible only after a merger. However unlike this paper, Inderst and Shaffer consider only two buyers and only two sellers; they do not consider which coalitions are stable when there are more than two buyers, when buyers are asymmetric, and when buyers have the choice of purchasing from more than two firms. But in some respects their paper is more general. They show that when the upstream firms’ product position is endogenous, the likelihood of a merger between retailers decreases upstream product differentiation which exacerbates the welfare costs associated with a retailer merger. They also show that when upstream firms are restricted to linear sales contracts, retailer mergers reduce the double marginalization associated with linear contracts, which reduces the welfare costs associated with a retailer merger. And finally, they show that when buyers have more bargaining power, the return to exclusive purchase commitments is diminished, which diminishes the welfare costs of a retail merger.

Snyder (1996, 1998) and Marvel and Yang (2008) show buyer groups can impact seller rivalry even without exclusive purchase agreements. Snyder (1996, 1998) shows that sellers’ ability to tacitly coordinate falls with buyer size (see also Tyagi, 2001). Marvel and Yang (2008) show that when buyers have private information (firms know the distribution of preferences within buyer groups, but not individual buyers’ preferences) and firms use non-linear pricing when making their price offers to groups, but use linear pricing when making offers to individuals, then buyers groups can increase buyer surplus. Buyer groups increase buyer surplus because they induce firms to compete in nonlinear price schedules, which makes competition more intense.

There is also a growing empirical literature on buyer groups. Three empirical papers and one experimental paper examined the impact of buyer size on buyer surplus (or price). These are Chipty (1995), Sorensen (2003), Ellison and Snyder (2001), and Normann et al. (2007). In addition, Moore and Newman (1993); Grabowski (1988); Dranove (1989); and Grabowski et al. (1992) look at the impact of drug formularies on price. All of these papers find evidence of a positive relationship between buyer size and buyer surplus, though, consistent with theory, the experimental paper (Normann et al., 2007) finds this effect only when seller’s marginal costs are increasing. Only Ellison and Snyder (2001) look at the role of competition on the buyer size premium. They find that buyer size is most valuable when buyers face more than one seller, which is consistent with theory presented here (as well as with Snyder 1996, 1998, and Marvel and Yang, 2008).

The paper is organized as follows. I present the model in Section 2. In Section 3, I characterize equilibrium buyer groups when there are just two sellers. I show that the grand coalition is a coalition-proof equilibrium, but that other payoff-equivalent equilibria exist with other configurations of buyer groups, including equilibria with arbitrarily many buyer groups. Each buyer’s surplus is constant across all equilibria when transfers are feasible, but only total buyer surplus is constant across equilibria in the absence of transfers. In Section 4, I contrast my results with related theoretical work on bundling in auctions. In Section 5, I consider equilibrium of the game when buyer groups use majority rule to choose the seller. In Section 6, I characterize equilibrium buyer groups when there are $n$ sellers. I show that with three or more sellers, the grand coalition is never an equilibrium, and when transfers are not feasible, every coalition-proof equilibrium has at least one group for each pair of sellers. With transfers and symmetric preferences, one equilibrium has at least $n(n - 1)/2$ buyer groups (one for each pair of firms) and I show that the payoffs in this equilibrium are unique when there are three sellers. In Section 7, I offer some concluding remarks.
2. The model

Consider $n$ differentiated sellers each selling a single product to a continuum of buyers who each demand at most one unit of the good. Each buyer’s valuations for the $n$ sellers’ products are denoted by the vector $v$, and the distribution of these valuations over the set of all buyers is denoted by the cumulative distribution function $F(v)$, where $v = (v_1, \ldots, v_n)$.

Without loss of generality, I assume $v_1 \geq v_2 \geq \cdots \geq v_n$ for a positive measure of buyers. The following definitions are useful:

**Definition 1.** Buyers preferences are **unanimous** if for all buyers $v_1 \geq v_2 \geq v_i, \forall i = 3, \ldots, n$. Buyers preferences are **symmetric** if $F(v') = F(v)$ whenever $v'$ is a simple permutation of $v$.

Buyers with unanimous preferences agree on their top choice and on their second choice, but may differ in their valuations and may differ in their ranking of the other products. Note that symmetry implies $\int v_1 \, dF(v) = \int v_j \, dF(v), \forall i, j$. Also note that if buyers’ preferences are symmetric then clearly they are not unanimous.

I assume that the sellers have complete information about the buyers’ preferences. I also assume that the sellers have constant marginal cost of production $c$ and that the support of $F$ is bounded from below by $c$, that is, $v_j > c$ for all buyers and for all $j = 1, \ldots, n$.

I assume that every buyer group is governed by a contractual agreement that (i) specifies transfer payments to, or from, each member, (ii) binds each member to purchase the product selected by the group at the group price, and (iii) binds the group to purchase exclusively from just one seller. I assume the group purchases from the seller that maximizes the total ex post surplus of its members. However for comparison, I also consider the case in which buyer groups use majority rule as their decision-making mechanism.

Each buyer group is completely defined by its membership, which I assume can be represented by a cumulative distribution function, $G^i(v)$, and by its associated transfer payment function, $t^i(v)$, which specifies the potentially negative transfer payment each member receives from the group. For convenience, I assume each group consists of a non-zero measure of buyers, i.e., $\lim_{v \to -\infty} G^i(v) > 0$. For comparison, I also consider the case in which transfers are not possible (i.e., $t^i(v) = 0$ for all $v$ for all $i$). A buyer group is **trivial** if it has zero measure or if it yields the same consumption and payoffs for its members as purchasing independently.

**Definition 2.** A potential buyer group, denoted by $(G^i(v), t^i(v))$, satisfies **budget balance** if $\int t^i(v) \, dG^i(v) = 0$ and is **feasible** if it satisfies budget balance and if $G^i(v) \leq F(v)$, $\forall v$. A set of $m$ buyer groups is **feasible** if each buyer group in the set is feasible and if $\sum_{i=1,\ldots,m} G^i(v) \leq F(v), \forall v$.

I consider a simple, stylized game of group formation in which buyers choose which type of group, $(G^i(v), t^i(v))$, to join from the set of feasible groups, and groups form only if there are enough buyers to meet the stipulated membership, $G$.

The timing of the game is as follows:

1. **Buyer Group Formation Stage**
   - (a) Buyers simultaneously choose which feasible buyer group, $(G^i(v), t^i(v))$, to join or, if none, to become an independent buyer.\(^{10}\)
   - (b) A buyer group forms if and only if the buyers that choose to join the group are sufficient to meet the membership objective of the group, $G^i$. Excess buyers who try to join a group which has already met its stipulated membership requirements become independent buyers. Buyers who try to join a group which does not meet its stipulated membership also become independent buyers. Each buyer group that forms is assigned a unique index $i$.
   - (c) If a buyer group forms, transfer payments are paid immediately.

2. **Pricing Stage** – Sellers simultaneously make take-it-or-leave-it price offers to each independent buyer, $p_j(v)$, and to each buyer group, $p_j^i$.

3. **Purchasing Stage** – Independent buyers and buyer groups make their purchase decisions, given sellers’ prices. If a product is purchased, each buyer pays the agreed price to the chosen firm and the buyers receive the good.

In the first stage, only feasible groups form (because only feasible groups are in the choice set) and any set of buyer groups that forms is feasible because each buyer can only join one group. Also, the distributions and transfer payments of the set of $m$ buyer groups that forms the first stage fully characterizes the strategies of buyers (the choice of which group to join), so $G^i(v)$ and $t^i(v), \forall i = 1, \ldots, m$ conveniently represents the first-stage strategies as well as the first-stage outcome.

\(^{10}\) Strictly speaking buyers choose from a set containing arbitrarily many copies of each feasible buyer group, and this set is not countable, so indexing each feasible group by $i$ is invalid.
In the pricing stage, given any feasible set of buyer groups, the sellers make take-it-or-leave-it price offers to each independent buyer and each buyer group. Each seller's price offer maximize its profit given the other sellers' equilibrium price offers and the equilibrium strategies of buyers.

In the purchasing stage, each independent buyer purchases from the seller that maximizes its surplus. Each buyer group purchases exclusively from the seller that maximizes the total surplus of its members. If the buyer group is indifferent it mixes with equal probability between its preferred sellers.

Because this is a game of perfect information, it is straightforward to use backward induction to rollback the payoffs in the purchasing and pricing stages to compute the payoffs in the group formation stage as a function of each buyer's valuations, \( \mathbf{v} \), and the distribution of buyers' valuations, \( G_1(\mathbf{v}) \), in the group which they joined.

An equilibrium outcome of the group formation game is fully characterized by the set of \( m \) feasible buyer groups which the buyers choose to join. I assume the equilibrium at the group formation stage is individually rational and coalition-proof, which I define as follows:

**Definition 3.** A feasible set of buyer groups, \((G_1(\mathbf{v}), t_1(\mathbf{v}))\), \(\forall i \in \{1,\ldots,m\}\), is individually rational if no buyer could earn a strictly higher payoff purchasing as an individual buyer and if at least some buyers (a positive measure of buyers) are strictly better off in the group than acting as individual buyers. A feasible set of buyer groups, \((G_1(\mathbf{v}), t_1(\mathbf{v}))\), \(\forall i \in \{1,\ldots,m\}\), is coalition-proof if there exists no set of buyers, with strictly positive measure, and an associated buyer group, \((G_{m+1}(\mathbf{v}), t_{m+1}(\mathbf{v}))\), such that if these buyers switched to join the new group \(G_{m+1}(\mathbf{v})\), then every subset of buyers in \(G_{m+1}(\mathbf{v})\), with strictly positive measure, would earn a strictly higher surplus.

This definition of coalition-proof is simpler then the standard definition given in Bernheim et al. (1987) because the payoffs for each buyer are independent of the equilibrium strategies of buyers outside the buyer's group, so I don't need to define coalition-proof recursively. While I could have applied their definition (which applies to more general games, not just games of group formation), I would have had to introduce more notation for the payoff functions and strategic decisions in the group formation game.

Summarizing, a coalition-proof subgame perfect Nash equilibrium (CP-SPNE) of the game is a set of buyer groups, a set of price offers associated with every potential buyer group, and a set of purchase decision rules associated with every potential buyer group and every potential set of price offers, such that (1) the price offers and purchasing decision rules are a subgame perfect Nash equilibrium of the pricing and purchasing subgames for all feasible sets of buyer groups, (2) no buyer is strictly better off purchasing as an individual, and (3) no new buyer group exists which gives every member of the group strictly higher surplus.

### 3. Coalition-proof subgame perfect Nash equilibria with two sellers

The equilibrium behavior of buyer groups is easiest to understand when there are just two sellers. For simplicity of exposition, I refer to the two sellers as Firm A and Firm B, and refer to buyers' valuations, \( \mathbf{v} \), as \((v_A, v_B)\). In Section 6 I consider \( n \) sellers.

First, consider Stages 2 and 3 of the game when no buyer groups form.12

**Lemma 1.** When no buyer groups form, the equilibrium price offers made to an individual buyer with valuations \( v_A \) and \( v_B \) are \( \max\{c, c + v_A - v_B\} \) from Firm A and \( \max\{c, c + v_B - v_A\} \) from Firm B. In equilibrium, each buyer purchases from Firm A if \( v_A > v_B \) and purchases from Firm B if \( v_B > v_A \). If \( v_A = v_B \), then both sellers' prices are \( c \) and the buyer mixes between purchasing from Firm A and Firm B. Each buyer's surplus is \( \min\{v_A, v_B\} - c \).

So when \( c = 1 \), a buyer with valuations (3, 6) buys from Firm B at a price of 4 (and rejects Firm A's offer of a price of 1) while a buyer with valuations (9, 4) buys from Firm A at a price of 6 (and rejects Firm B's offer of a price of 1).

Lemma 1 implies Firm A's surplus is \( \int \max\{v_A - v_B, 0\} dF(\mathbf{v}) dv \) and Firm B's surplus is \( \int \max\{v_B - v_A, 0\} dF(\mathbf{v}) dv \) and, more importantly, that sellers only earn rents from buyers with heterogeneous preferences. If a buyer has homogeneous preferences, or makes purchasing decisions as if they had homogeneous preferences, then the sellers earn zero profits.

Let
\[
\overline{v}_j \equiv \frac{\int v_j dF(\mathbf{v})}{\int dF(\mathbf{v})}, \quad j = A, B
\]

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11 Buyers can do even better if they are free to make any strategic commitment. But a buyer group is a simple, practical, and potential very credible commitment. The credibility may be due to the transactions costs of renegotiation a contract with many participants. See Perotti and Spier (1993) and Hart and Moore (1995) for applications of this argument to debt contracts.

12 I assume that each individual buyer can purchase from only one of the two firms, so the firms are not competing in a menu auction. If buyers could split their purchases and menu offers were feasible, Lemma 1 would still hold under the truthful Nash equilibrium refinement (see Bernheim and Whinston, 1986).
denote the average valuation for Firm $j$’s product within the grand coalition of all buyers, and let

$$
\bar{v}_j^i = \frac{\int v_j dG^i(v)}{\int dG^i(v)}, \quad j = A, B
$$

denote the average valuation for Firm $j$’s product within buyer group $i$. Then clearly the equilibrium price offers for a buyer group with mean valuations $\bar{v}_A^i$ and $\bar{v}_B^i$ are the same as those of an individual with these valuations. Moreover,

**Lemma 2.** Joining a buyer group which satisfies $\bar{v}_A^i = \bar{v}_B^i$ and $t^i(v_A, v_B) = 0$, $\forall v_A, v_B$ is always individually rational.

Lemma 2 holds because the equilibrium the price offers (the average, or per member, price) are equal to marginal cost and the buyer group will mix between the two sellers, so each buyer’s surplus is $(v_A + v_B)/2 - c$, which is strictly greater than $\min(v_A, v_B) - c$. So all buyers in this buyer group are strictly better off, except at most a set of buyers of measure zero for whom $v_A = v_B$, who are only weakly better off. Using Lemma 2, it is easy to characterize the set of CP-SPNE when transfers are not feasible.

**Proposition 1.** In the absence of transfer payments, a feasible set of $m$ buyer groups is individually rational and coalition proof and represents a CP-SPNE, if and only if $\bar{v}_A^i = \bar{v}_B^i$, $\forall i = 1, \ldots, m$, and there is no positive measure of independent buyers that could form a new buyer group satisfying $\bar{v}_A^{n+1} = \bar{v}_B^{n+1}$. If buyers’ preferences are unanimous, then the unique equilibrium has no buyer groups.

For example, when $c = 1$ and consumers have valuations $(3, 6)$ and $(9, 4)$, the buyers’ surplus when independent is 2 and 3 respectively, but if one of each type formed a buyer group, the group would buy from Firm $A$ at a price of 2, but this would make the former buyer worse off. Joining this group isn’t individually rational. But if three of the latter buyers formed a buyer group with five of the former buyers, then $\bar{v}_A^i = \bar{v}_B^i$, so this group would receive price offers of 1 from both firms and mix between buying from Firm $A$ and Firm $B$. In this case both types are better off, so joining this group is individually rational.

Lemma 2 and Proposition 1 generalize the numerical example given in the Introduction by allowing for a more general distribution of buyer valuations and a more general composition of buyer groups. As in the example, Lemma 2 emphasizes the ability of buyer groups to use their composition to commit to being indifferent between the two sellers. Because they value the products equally, it is credible to buy from the lowest priced seller and mix when the sellers’ prices are equal.

Note that the remaining independent buyers strictly prefer to be in a buyer group. So the assumption that the group still forms even when too many buyers try to join is important. If assignment to the group is random, then in the group formation stage every buyer tries to join and only some are able to. If assignment to the group is deterministic, then the omitted buyers weakly prefer to remain independent. However, in either case there does not exist a coalition that can increase its members surplus by forming a new group.

Finally, note that all CP-SPNE have the same total surplus for buyers, for Firm $A$, and for Firm $B$. The only difference between the equilibria is the number and composition of the groups that form and which buyers are able to reap the benefits of joining groups when some buyers remain independent.

When transfer payments are feasible, the set of buyer groups that can form in equilibrium is richer. In particular individual rationality can be satisfied for buyer groups that do not satisfy $\bar{v}_A^i = \bar{v}_B^i$. However, while there are many transfer payment schedules that satisfy individual rationality, the group’s transfer payments must also be coalition proof with respect to any subset of the group members. The following lemma demonstrates that transfer payments exist which satisfy individual rationality and preclude deviations by any subcoalition of the group.

**Lemma 3.** For any distribution of buyers, $G^i(v)$, there exists transfer payments, $t^i(v)$, that make the group individually rational and coalition proof with respect to deviations by any subcoalition of the group. Specifically,

(i) when $\bar{v}_A^i > \bar{v}_B^i$, then the unique transfer payment schedule is $t^i(v_A, v_B) = \bar{v}_A^i - \bar{v}_B^i + v_B - v_A$ for all $v_A, v_B$ and the unique individual buyer’s surplus is $v_B - c$;
(ii) when $\bar{v}_A^i > \bar{v}_B^i$, then the unique transfer payment schedule is $t^i(v_A, v_B) = \bar{v}_B^i - \bar{v}_A^i + v_A - v_B$ for all $v_B, v_A$ and the unique individual buyer’s surplus is $v_A - c$;
(iii) when $\bar{v}_A^i = \bar{v}_B^i$, the transfer payments are $t^i(v_A, v_B) = \theta(v_A - v_B)$ where $\theta$ is any value in the interval $[-1/2, +1/2]$. Each individual buyer’s surplus is $(1/2 + \theta)v_A + (1/2 - \theta)v_B$.

For example, if an equal number of buyers with valuations $(3, 6)$ and buyers with valuations $(9, 4)$ form a buyer group, the group buys from Firm $A$ at a price of 2 and the latter buyers each pay a transfer of 4 to the former buyers. The former buyers’ surplus is 5, which is $v_B - c$, and the latter buyers’ surplus is 3, which is also $v_B - c$. The transfer payment is unique because the former buyers are scarce.

Intuitively, when $\bar{v}_A^i > \bar{v}_B^i$ (i.e., line i of Lemma 3) members of group $i$ purchase from Firm $A$ at a price $c + \bar{v}_A^i - \bar{v}_B^i$ and get surplus $v_A - c - \bar{v}_A^i + \bar{v}_B^i + t^i(v_A, v_B)$. Group members for whom $v_A > v_B$ must get at least $v_B - c$ in surplus or joining
the group isn’t individually rational, and clearly \( t^i(v_A, v_B) = v_A^i - v_B^i + v_B - v_A \) gives them exactly this surplus. However it also cannot be the case that any of these group members earn strictly higher surplus since otherwise a subcoalition can form that excludes some of these buyers and divides their surplus among all the remaining members. A similar argument implies that group members for whom \( v_B > v_A \) cannot get more surplus than \( v_B - c \) since they too could be profitably excluded. So the transfer payments are uniquely defined.

The following is an intermediate step in characterizing the set of CP-SPNE. It shows that, like individuals, buyer groups can also be made better off by merging as long as the groups have heterogeneous preferences.

**Lemma 4.** A coalition of buyers consisting of all the members of two feasible buyer groups earns strictly greater total surplus joining a new combined group if the two buyer groups would have otherwise purchased from different firms and has no effect on total surplus if the buyer groups would have otherwise purchased from the same firm.

Lemma 4 shows that a set of buyer groups cannot be coalition proof if two groups always purchase from different sellers because transfers clearly exist that make every buyer better off if the two groups merge. Proposition 2 proves that this condition is also sufficient. That is, as long as no profitable buyer-group merger exists (and no profitable combination of independent buyers exists), then there exists transfer payments that make any set of buyer groups a coalition-proof subgame perfect equilibrium.

**Proposition 2.** When transfer payments are feasible, a CP-SPNE with non-trivial buyer groups exists if and only if preferences are not unanimous. Given preferences are not unanimous, a set of feasible buyer groups is a CP-SPNE, if and only if

(i) \( v_A > v_B \); every buyer group weakly prefers Product A to B, i.e. \( v_A^i \geq v_B^i \), and every buyer with valuation \( v_B > v_A \) joins a buyer group. Each buyer’s surplus (including independent buyers) is equal to \( v_B - c \).

(ii) \( v_A < v_B \); every buyer group weakly prefers Product B to A, i.e. \( v_B^i \geq v_A^i \), and every buyer with valuation \( v_A > v_B \) joins a buyer group. Each buyer’s surplus (including independent buyers) is equal to \( v_A - c \).

(iii) \( v_A = v_B \); every buyer group is indifferent between A and B, i.e. \( v_A^i = v_B^i \), and every buyer joins a buyer group. Buyers’ average buyer surplus is equal to \( (v_A + v_B)/2 - c \). Each buyer’s surplus is not uniquely defined, but \( 3p \in [0, 1] \) such that each buyers’ surplus is equal to \((pv_A + (1-3p)v_B)/2 - c\).

For example, consider again \( c = 1 \) and consumers with preferences \((3, 6)\) and \((9, 4)\). When \( v_A > v_B \), i.e., they are sufficiently many of the latter, all of the buyers of the former type join buyer groups and all earn a surplus of 5 whereas they would have earned only a surplus of 2 if the bought independently. Buyers of the later type earn a surplus of 3, the same as they earned if they bought independently. If there are sufficiently many of the former \((v_B > v_A)\), then all of the latter type join buyer groups and earn a surplus of 8 while the former type earn a surplus of 2.

Intuitively, if \( v_A > v_B \), then there are a surplus of buyers for whom \( v_A > v_B \) who are available to join a group with buyers for whom \( v_B > v_A \). Because they are in excess supply, their surplus in a coalition-proof equilibrium must be equal to their outside option, which is \( v_B - c \). Buyers for whom \( v_B > v_A \) also capture their outside option, which is to form a new group with other buyers for whom \( v_A > v_B \), in which case their surplus is \( v_B - c \). So when \( v_A > v_B \), in all coalition-proof subgame perfect equilibria, each buyer’s unique surplus is \( v_B - c \).

While each individual buyer’s surplus is unique when \( v_A > v_B \) (or \( v_B > v_A \)), each individual buyer’s surplus is not unique when \( v_A = v_B \). The total buyer surplus, which is unique, can be divided arbitrarily between buyers who prefer Product A to Product B and buyers who prefer Product B to Product A. But within these two groups, buyers are treated symmetrically.

Proposition 2 clearly implies the following corollary.

**Corollary 1.** The grand coalition is a CP-SPNE.

Of course the grand coalition is not the only equilibrium. Proposition 2 implies that there are many other equilibrium coalition structures all with the same individual and total buyer surplus as the grand coalition. One obvious way to construct new equilibria is to divide the grand coalition into smaller groups. For example, clearly \( C^i(\nu) = F(\nu)/m, i = 1, \ldots, m \), is an equilibrium buyer group structure for all \( m \).

Another way to construct new equilibria is to remove some buyers from buyer groups. When \( v_A > v_B \), buyers for whom \( v_A > v_B \) must be indifferent between joining a group and not joining. The equilibrium buyer groups must satisfy \( v_A^i \geq v_B^i \) for all \( i \), but payoff-equivalent equilibria exist in which all, none, or only some of the remaining buyers join buyer groups. If more join, the group price rises, but the average size of the transfer from buyers for whom \( v_A > v_B \) to buyers for whom \( v_A < v_B \) falls, so each buyer’s surplus remains the same.

Comparing Proposition 2 to Proposition 1, it is clear that introducing transfers does not change total surplus or the division of total surplus between buyers and sellers. Transfers only change the allocation of surplus across buyers. For example, suppose the two thirds of the buyers have valuations \( v_A = 6 \) and \( v_B = 3 \), the remaining one third have valuations \( v_A = 3 \) and \( v_B = 6 \), and assume \( c = 0 \). Then, \( v_A = 5 \) and \( v_B = 4 \). Absent transfer payments, every CP-SPNE has a measure 1/3 of the former type and a measure 1/3 of the latter type joining groups. Those who join groups each get a surplus of
gets a surplus of \((1/3)3 = 1\), which is a surplus of \(3\) on each sale to an independent buyer, and Firm \(B\) gets a surplus of zero. Total surplus is \((2/3)4.5 + (1/3)3 + (1/3)3 = 5\). With transfer payments, the grand coalition is supported by a CP-SPNE. Firm \(A\) gets a surplus of \(v_A - v_B = 1\) and Firm \(B\) gets a surplus of zero. The buyers of the former type get a surplus of \(4.5\) (6 less a transfer of 1.5) and the buyers in the latter group get a surplus of \(5\) (2 plus a transfer of 3), so total surplus is \((1/3)2 + (2/3)5 + 1 = 5\). This holds more generally and is stated in the following Corollary.

**Corollary 2.** The aggregate buyer surplus and aggregate seller surplus are not changed by the introduction of transfer payments. However, the allocation of surplus across buyers may be changed.

While total surplus is unchanged, notice that introducing transfers eliminates the inequity that can arise between identical buyers some of who are able to join a buyer group and some of whom cannot. While this is appealing, it comes at the cost of treating members of the buyer group asymmetrically. That is, some buyers in the group pay more for the good than others.

Finally, some of the implications of Proposition 2 may appear counter-intuitive to readers. One is that all buyers seem to purchase from the same seller. Clearly this is not what is observed empirically, and as importantly, if it were, the implication is that competition would not survive and the surviving firm would acquire significant market power. However, Proposition 2 does not imply that all buyers purchase from the same seller. In fact, the CP-SPNE in Proposition 1 is also a CP-SPNE in Proposition 2. The total buyer surplus in all CP-SPNE in Proposition 2 is the same as the total buyer surplus in the CP-SPNE of Proposition 1. Randomizing between sellers is still an equilibrium outcome when transfers are feasible (whether or not \(v_A = v_B\)). Also if groups form when the buyers have noisy signals of the preferences and then make their purchase decisions after learning those preferences, competition might survive even with asymmetric buyer groups. Perhaps more importantly, the constant marginal cost assumption is unrealistic in many of the healthcare and manufacturing examples mentioned earlier. With strictly increasing costs, buyer groups could still intensify price competition, but even groups which preferred Firm \(A\)’s product, i.e., \(v_A > v_B\), might form and purchase from Firm \(B\) in equilibrium.

Another implication is that an individual buyer’s surplus appears to be very sensitive to the composition of the group. When no transfers are feasible, a small change in the preferences of a small mass of consumers could cause the group to purchase exclusively from one seller and violate individual rationality for much of the group. However, this is simply emphasizing that the randomness of the groups decision generates wide fluctuations in payoffs, whether that randomness reflects changes in the groups composition or a coin toss. A related concern arises when transfers are feasible, but not when transfers can be made contingent on the groups purchase decision and are chosen to make consumers indifferent with respect to the group’s decision, which can easily be done without affecting individual rationality.

A final implication is that when transfers are feasible and \(v_A > v_B\), none of the buyers for whom \(v_A > v_B\) earn any additional surplus from joining a buyer group – their individual rationality constraints bind. This is an implication of coalition proofness, but it nevertheless seems to conflict with empirical evidence. However, this is not the case when we consider three or more sellers. In that environment, buyers outside option is to join a different group. This means that the individual rationality constraint is much less likely to bind.

And most importantly, buyer groups are likely to form for the purchase of multiple products. It may be the case that the rents buyers capture from joining a group are zero for some purchases and positive for others. This is a reasonable interpretation of HMO’s and PPO’s drug formularies which extend across many, many therapeutic classes of drugs. Uncertain preferences would have a similar effect.

### 4. Bundling in auctions

My analysis is closely related to the literature on bundling in auctions. In particular, Palfrey (1983) shows that an auctioneer selling \(J\) indivisible, non-rival goods to \(n\) privately informed buyers benefits from bundling those products together and committing to sell the bundle to a single buyer. Specifically, Proposition 2 above is closely related to Palfrey’s Theorem 2. Palfrey’s Theorem 2 showed that with exactly two buyers, and assuming independently and identically distributed valuations, the seller always maximizes its profit by bundling all of its products and selling them in a single auction as opposed to selling each product in a separate auction.

Of course, the two papers differ semantically. The buyers in this paper, treated collectively, are Palfrey’s seller, while the sellers in this paper are Palfrey’s buyers. The models also differ in five other important ways. First, my buyers’ group decisions must be coalition proof while Palfrey’s seller’s bundling decision simply maximizes total seller surplus. Second, I consider a continuum of heterogeneous buyers as opposed to a discrete number of heterogeneous products. Third, Palfrey allows the seller to choose between a first and second price auction (and Avery and Hendershott, 2000, and Armstrong, 2000 consider even more general selling mechanisms) while I take the market mechanism, a first-price auction, as given. Fourth, my buyers have complete information about sellers’ costs (and each other’s valuations) when choosing their buyer group.

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13 While also related to the general literature on multiproduct selling (see Armstrong and Rochet, 1999), it is more closely related to the literature on auctions since the assumption that sellers are capacity constrained in an auction is analogous to my assumption that buyers have unit demands.
strategy, while Palfrey’s seller knows only the distribution of his buyers’ valuations when choosing its bundling strategy.\footnote{Because the selling format in Palfrey (1983) is restricted to a first or second price auction without reservation prices, his assumption that buyers are privately informed about their valuations is only relevant to the bundling decision, not to the selling mechanism.} Finally, I relax the assumption that valuations are independently and identically distributed.

Despite the differences, clearly my results are closely linked to those in the bundling-in-auctions literature. In particular, since the grand coalition is optimal when the valuations are common knowledge, in Palfrey’s model, regardless of the buyers’ valuations, bundling every product must be optimal when the auctioneer knows only the distribution of buyer’s valuations (Palfrey, 1983, Theorem 2). Thus Corollary 1 implies we can prove Palfrey’s result without the independently and identically distributed valuations assumption, at least for a continuum of products.

**Proposition 3.** When selling a continuum of products to two buyers, \( A \) and \( B \), in a first-price auction, bundling every product together strictly increases profits for any distributions of buyer valuations, \( F(v_A, v_B) \) which is not unanimous (i.e., neither buyer’s willingness to pay is strictly higher than the others for every product).

5. Coalition-proof subgame perfect Nash equilibria under majority rule

In this section I ask what would happen under alternative group decision-making mechanisms. The most plausible alternative to surplus maximization is majority rule. I find that when transfers are feasible and buyer groups are constrained to use majority rule, the set of equilibrium buyer groups that can be supported under majority rule is a subset of those that can be supported under total surplus maximization, however total buyer surplus is unchanged. However, I find that when transfers are not feasible and buyer groups are constrained to use majority rule, then the set of equilibrium buyer groups that can be supported under majority rule often includes equilibria with both higher and lower total buyer surplus than those that can be supported under total surplus maximization, and in some circumstances, total buyer surplus will be strictly lower.

**Proposition 4.** When transfers are feasible and can be made contingent on the purchase decision, the CP-SPNE under majority rule are the same as under total buyer surplus maximization.

When transfers can be made contingent on the purchase decision, then CP-SPNE is payoff equivalent to a CP-SPNE in which \( \bar{v}_i^A = \bar{v}_i^B, \forall i \), and given this, transfers that are contingent on the purchase decision can always be designed to make every member of the group indifferent between the two products without changing the expected transfer payment. So any CP-SPNE is payoff equivalent to a CP-SPNE in which \( \bar{v}_i^A = \bar{v}_i^B, \forall i \), in which all buyers are individually indifferent between \( A \) and \( B \) (i.e., \( v_A + t_A^i(v_A, v_B) - c = v_B + t_B^i(v_A, v_B) - c \)), and in which individual rationality is satisfied (i.e., \( (v_A + v_B)/2 + (t_A^i(v_A, v_B) + t_B^i(v_A, v_B))/2 \geq \min(v_A, v_B) - c \)). And given these groups, total surplus maximization and majority rule are equivalent.

**Proposition 5.** When transfers are not feasible, the total buyer surplus in a CP-SPNE under majority rule may be either higher or lower than the total buyer surplus maximization in a CP-SPNE under surplus maximization.

When transfers are not feasible (or cannot be made contingent on the purchase decision), then each buyer group is indifferent if only if \( \int_{v_A > v_B} dG^i(v) = \int_{v_A < v_B} dG^i(v) \). Suppose there are more buyers for whom \( v_A > v_B \) than buyers for whom \( v_B > v_A \), i.e., \( \int_{v_A > v_B} dF(v) \geq \int_{v_B > v_A} dF(v) \). Then a coalition made up of every buyer for whom \( v_B > v_A \) and an equal number of buyers for whom \( v_A > v_B \) is a CP-SPNE. And clearly there are many other CP-SPNE in which this group is divided up into smaller groups, each satisfying \( \int_{v_A > v_B} dG^i(v) = \int_{v_A < v_B} dG^i(v) \). Total buyer surplus is highest when the buyers for whom \( v_A > v_B \) that join the group are those with the strongest preferences, \( v_A - v_B \). That is, total buyer surplus is maximized when \( \bar{v}_A^i - \bar{v}_B^i \) is maximized, conditional on \( \int_{v_A > v_B} dG^i(v) = \int_{v_B > v_A} dG^i(v) \). Total buyer surplus is the same if \( \bar{v}_A^i = \bar{v}_B^i \). And total buyer surplus in minimized when \( \bar{v}_A^i - \bar{v}_B^i \) is minimized, conditional on \( \int_{v_A > v_B} dG^i(v) = \int_{v_B > v_A} dG^i(v) \).

Finally, if \( \bar{v}_A > \bar{v}_B \) and \( \int_{v_A > v_B} dF(v) \geq \int_{v_B > v_A} dF(v) \), so the average buyer prefers Product \( A \) to Product \( B \) and there are more buyers for whom \( v_A > v_B \) than buyers for whom \( v_B > v_A \), then an equilibrium always exists in which total buyer surplus is strictly higher under majority voting. On the other hand, if \( \bar{v}_A < \bar{v}_B \) and \( \int_{v_A > v_B} dF(v) \geq \int_{v_B > v_A} dF(v) \), so (perhaps counter intuitively) the average buyer prefers Product \( B \) to Product \( A \) even though there are fewer buyers who prefer Product \( B \) to Product \( A \), then in any equilibrium, total buyer surplus will be strictly lower under majority rule than under total surplus maximization.
6. Coalition-proof subgame perfect equilibria with \( n \) sellers

With \( n \) sellers the grand coalition is never a CP-SPNE. The grand coalition is only coalition proof when buyers’ preferences are unanimous and transfer payments are feasible, but in this case grand coalition is not individually rational since it cannot make any buyers strictly better off.

To understand why the grand coalition is not coalition proof when preferences are not unanimous, consider symmetric buyers’ preferences. In this case the grand coalition is not coalition proof because all of the buyers whose highest and second highest valuations are for Firm \( j \)’s and Firm \( k \)’s products would be strictly better off leaving and forming a new group. This deviating group clearly prefers Firm \( j \)’s and Firm \( k \)’s products to all other products, and by symmetry is indifferent between these products, so Firm \( j \) and Firm \( k \) would both offer a price equal to marginal cost the average surplus of the groups members from increase from \( \int \frac{\sum_{v} dv}{n} / \int dv = c \) (since the grand coalition randomizes across all \( n \) suppliers) to \( \int G_{jk}(v) / \int G(v) = c \) (since the deviating group randomizes between its top two choices), where \( G_{jk}(v) \) is the distribution of \( v \) conditional on products \( j \) and \( k \) have the highest valuations (i.e., \( \min(v_i, v_j) \geq v_k, \forall k \neq i, j \)). This is clearly a strictly positive increase.

So what are the CP-SPNE? First, suppose transfers are not feasible. With \( n \) sellers, buyers’ surplus is higher if groups try to induce competition between pairs of firms, not all \( n \) firms at once. Consider a set of \( n(n-1)/2 \) buyer groups, one for each pair of firms, \( jk \), each of which consists of all the buyers whose highest valuation is for Firm \( j \)’s product and second highest valuation is for Firm \( k \)’s product and all the buyers whose highest valuation is for Firm \( k \)’s product and second highest valuation is for Firm \( j \)’s product. By symmetry, \( v_{jk}^p = v_k^p \) for all \( jk \), so when the transfer payments are equal to zero, every buyer in group \( jk \) earns a surplus of \( (v_j + v_k)/2 - c \).

**Proposition 6.** When transfers are not feasible and \( F \) is symmetric, then a set of \( n(n-1)/2 \) buyer groups each containing only those buyers whose highest two valuations are \( v_i \) and \( v_j \) for some \( i \) and some \( j \), and transfer payments equal to zero, is an CP-SPNE and all CP-SPNE are payoff equivalent and have at least \( n(n-1)/2 \) buyer groups.

Clearly Proposition 6 also characterizes an equilibrium when transfers are feasible. In order to analyze CP-SPNE for more general distributions, and with transfers, I assume that \( F \) is continuously differentiable and I introduce the following notation.

**Definition 4.** A type \( j > k \) buyer is one for whom \( v_j > v_k > v_i, \forall i \neq j, k \). A type \( j > k \) buyer group is one for whom \( v_j > v_k > v_i, \forall i \neq j, k \). A type \( jk \) buyer (or buyer group) is one that is either a type \( j > k \) buyer (or buyer group) or a type \( k > j \) buyer (or buyer group).

Note that I can further simplify by focusing only on coalition-proof equilibria with just one type \( jk \) group, or at most \( n(n-1)/2 \) groups in all. That is, any CP-SPNE is payoff equivalent to a CP-SPNE with at most \( n(n-1)/2 \) buyer groups, one of each buyer group type (i.e., one each in the set \( M = \{jk: j, k \in \{1, \ldots, n\}, j < k \} \)). This is true for two reasons. First, a version of Lemma 4 applies with \( n \) sellers as well, so a type \( j > k \) group and a type \( k > j \) group cannot both exist in a CP-SPNE since the members would be better off joining together in a new group. This implies that I do not need to keep track of \( j > k \) and \( j \) \& \( k \) groups separately since they won’t coexist. And second, if there are more than one type \( j > k \) group, then they all can be combined into a single group without changing any of the transfer payments, purchase decisions, or each individual buyer’s surplus. So the set of groups obtained by combining like groups is feasible, individually rational, and coalition proof as long as the groups being combined are feasible, individually rational, and coalition proof. It is now straight-forward to prove the following.

**Proposition 7.** When transfer payments are not feasible and there exists buyers of type \( i > j, \forall i, j \), then every CP-SPNE has a minimum of \( n(n-1)/2 \) groups, one for each pair of firms.

The proof is simple and quite intuitive. Every type \( i > j \) and \( j > i \) buyer earns \( (v_i + v_j)/2 \) in a type \( ij \) group satisfying \( v_i = v_j \), which is higher payoff than they could earn in any other group. And at least some of these buyers exist. So a type \( ij \) group must exist because otherwise a subset of the type \( i > j \) and \( j > i \) buyers that could form a new group satisfying \( v_i = v_j \) and be strictly better off.

When transfers are feasible, I have only shown a similar result for more restrictive cases. The following lemma begins by showing that the equilibrium buyer groups in any CP-SPNE can be represented by a vector of prices, \( p_{jk} \in [0, 1] \).

**Lemma 5.** When \( n \) transactions are feasible, a set of buyer groups (with at most \( n(n-1)/2 \) buyer groups, one for each pair of firms) is a CP-SPNE if and only if there exists \( n(n-1)/2 \) prices, \( p_{jk} \in [0, 1] \), such that the distributions \( G_{ijk}(v) \) are defined by

\[
\max_{jk \in M} p_{jk} v_j + (1 - p_{jk}) v_k = c.
\]

and \( \forall jk \in M \), either (i) \( p_{jk}^p = v_k^p \cdot (i) p_{jk} = 1 \) and \( v_j \leq v_k \cdot (ii) p_{jk} = 0 \) and \( v_j^p \geq v_k^p \).
As in the case of two sellers, buyers must capture all of the value they add to the group when the equilibrium buyer group they join has a shortage of buyers with their preferences, but if the buyer groups are indifferent between two firms, then the surplus can be divided in other ways. However with more than two sellers, the division of the surplus is not arbitrary since buyers have the option of joining other groups to which they may also add value.

Lemma 5 can be used to show generally that a coalition-proof subgame perfect equilibrium exists. Proving existence involves finding a set of sharing rules, or prices, such that the market for buyer groups clears. The following proposition uses a fixed point argument to prove that such a set of sharing rules must exist.

**Proposition 8.** A CP-SPNE exists for all $F$ in the $n$-seller game.

Just as in the case of two sellers, many coalition-proof subgame perfect equilibria exist, including equilibria with arbitrarily many buyer groups. Buyer groups can be broken up into much smaller groups, and still be supported by the same sharing rules, without any impact on buyer surplus, as long as the groups’ mean valuations remain the same. Also if $p^j = 0$ (and similarly if $p^k = 1$), there clearly exist other equilibria in which some type $j > k$ buyers do not join groups, but these equilibria yield the same buyer surplus.

If $n = 3$, and $F$ is strictly increasing and symmetric, then Lemma 5 can also be used to prove the uniqueness of the equilibrium payoffs in Proposition 6.

**Proposition 9.** When $F$ is strictly increasing and symmetric and $n = 3$, in every CP-SPNE each firm’s profit is zero and each buyer’s surplus is $v_j + v_k - c$ where $v_j$ and $v_k$ are the buyer’s two highest valuations (i.e., $\min v_j, v_k \geq v_i, i \neq j, k$).

The uniqueness of the symmetric payoffs stands in sharp contrast to the results for $n = 2$. In that case there were many coalition-proof ways to divide the gains from buyer group formation. However, in that case each buyers’ outside option was purchasing independently, but when $n = 3$, some buyers’ best outside option is to join a different buyer group. For example a buyer with valuations $(100, 50, 49)$ is almost as valuable to a 13 group as to a 12 group. Moreover, in a coalition-proof equilibrium, similar buyers are treated equally so a superior outside option for some buyers helps raise other buyers’ surplus.

7. Conclusion

Buyer groups can utilize heterogeneity in members’ preferences and an exclusive purchase commitment to capture more surplus than their members could capture on their own. The buyer group acquires market power by grouping buyers with heterogeneous preferences together to create a single decision maker that is indifferent between two sellers’ products. This induces sellers to compete more aggressively; while it also leads to inefficient consumption, the expected gain from increased price competition exceeds the expected loss from consuming the wrong product.

I emphasize the role of the exclusive purchase agreement, without which buyer groups have limited power. While I simplified the analysis by ignoring buyer groups without exclusive purchase agreements, in an earlier version of the paper I argued that buyer groups could still make buyers better off even without the exclusive purchase commitment. However, this argument relied on firms playing an equilibrium in the menu auction stage that did not satisfy the truthful Nash equilibria refinement (see Bernheim and Whinston, 1986).

Buyer groups are inefficient because they use exclusive purchase agreements that lead to ex post inefficient trade. However, there are some obvious efficiency benefits of increasing price competition that the model ignores. In particular, the assumption that buyers have unit demand combined with the assumption that sellers can perfectly price discrimination eliminates the standard dead weight loss associated with seller market power. Understanding the welfare impacts of buyer groups and exclusive purchase agreements, is a promising area for future research.

Understanding buyer group formation is another important area for future research. I analyzed a very stylized model of group formation. Research on more realistic, sequential games of coalition formation has the potential to generate important insights about buyer group behavior.

Finally, I assumed that each buyer is quoted a buyer-specific price. In many applications sellers price uniformly across individual buyers. Analyzing uniform pricing is important because group formation affects the price offered to independent buyers. However, understanding coalition formation with uniform pricing is challenging because the distribution of independent buyers is endogenous and need not be continuous once groups form. Understanding the impact of uniform pricing on the incentives for coalition formation is an important area for future research.

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15 Marvel and Yang (2008) face this problem in their paper. However, they do not require that buyer groups be coalition proof. Instead they assume that the buyer groups formed are the ones that maximize buyer surplus provided that they are Pareto superior to having no buyer groups.
Appendix A

Proof of Proposition 1. By Lemma 2, if independent buyers could form a new buyer group satisfying $\bar{v}_A^i = \bar{v}_B^i$, they would, so a set of groups is coalition proof only if all independent buyers prefer Product $A$ to Product $B$, all independent buyers prefer Product $B$ to Product $A$, or $\bar{v}_A = \bar{v}_B$ and there are no independent buyers.

A set of groups is individual rational only if $\bar{v}_A^i = \bar{v}_B^i$, $\forall i$. Suppose instead that $\bar{v}_A^i > \bar{v}_B^i$ for some $i$. Then group $i$ will buy from Firm $A$ at the price $\bar{v}_A^i - \bar{v}_B^i + c$, so individual rationality implies

$$v_A - (\bar{v}_A^i - \bar{v}_B^i + c) \geq \min\{v_A, v_B\} - c,$$

which cannot hold for $v_B > v_A$. Similarly, if $\bar{v}_A^i > \bar{v}_B^i$ for some $i$, then individual rationality cannot hold for $v_A > v_B$. These conditions are also sufficient because every buyer’s surplus is equal to $\frac{v_A + v_B}{2} - c$ which is the highest possible surplus the buyer could capture in a deviating group.

Finally, if preferences are unanimous then forming a group cannot increase total surplus and is never individually rational (except for consumers with identical preferences).

Proof of Lemma 3. When $\bar{v}_A^i > \bar{v}_B^i$ (and by analogy when $\bar{v}_B^i > \bar{v}_A^i$), if a positive measure of buyers, $H(v)$, who prefer Product $A$ to Product $B$, earn surplus strictly greater than $v_B - c$, the surplus they capture as independent buyers, then for a sufficiently small, there exists a new group $G(v) = G(v) - aH(v)$ that satisfies $\bar{v}_A^i > \bar{v}_B^i$ and that offers strictly higher surplus. The excluded members still buy from Firm $A$ but their surplus strictly falls to $v_B - c$, and since the group still buys from Firm $A$, total surplus must be the same, so the new group must be strictly better off. That is, the members of the new group can divide the surplus of the excluded members of the old group and all be strictly better off. Since these buyers’ individual surplus must equal $v_B - c$ (individual rationality implies the surplus cannot be lower), it follows that

$$v_A + t^i(v_A, v_B) - (\bar{v}_A^i - \bar{v}_B^i + c) = v_B - c,$$

so $t^i(v_A, v_B) = \bar{v}_A^i - \bar{v}_B^i + v_B - v_A$ for all $v_A > v_B$.

Furthermore, if a positive measure of buyers who prefer Product $B$ to Product $A$, earn surplus strictly greater than $v_B - c$, then there must also be a strictly positive measure of buyers who prefer Product $B$ to Product $A$ and earn surplus strictly less than $v_B - c$ (since all buyers who prefer Product $A$ to Product $B$ earn exactly $v_B - c$). But in this case, clearly the later members could form a new group excluding the former members in which every member would be strictly better off.

When $\bar{v}_A = \bar{v}_B^i$, individual rationality implies that each buyer’s surplus as a member of the group and as an independent buyer satisfy

$$\frac{v_A + v_B}{2} + t^i(v_A, v_B) - c \geq \min\{v_A, v_B\} - c,$$

or

$$\max\left\{\frac{v_A - v_B}{2}, \frac{v_B - v_A}{2}\right\} + t^i(v_A, v_B) \geq 0.$$  

Clearly a wide range of transfer payments (including $t(v) = 0$) satisfies individual rationality.

However, the group is not coalition proof unless the members of every feasible subset of the group for which $\bar{v}_A = \bar{v}_B^i$ earn the same average surplus as they earn in group $i$, or

$$b \left[ \max\left\{\frac{v_A - v_B}{2}, 0\right\} + t^i(v_A, v_B) \right] + b^i(v_A, v_B) \geq 0,$$

or, by budget balance, $\int t^i(v_A, v_B)G(v) = 0$, or equivalently

$$\int t^i(v_A, v_B)G(v) = - \int v_a > v_b \int t^i(v_A, v_B)G(v),$$

for all $G(v) \subset G^i$ satisfying $\int (v_B - v_A)G(v) = 0$. However, this is feasible only if on the left-hand side $t^i(v_A, v_B)$ is proportional to $v_B - v_A$ when $v_B > v_A$ and on the right-hand side $t^i(v_A, v_B)$ is proportional to $v_B - v_A$ when $v_B > v_A$. This implies that the difference between each buyer’s surplus as a member of the group and as an independent buyer, (5), is also proportional to $v_B - v_A$ when $v_B > v_A$ and proportional to $v_B - v_A$ when $v_B > v_A$.

Proof of Lemma 4. Suppose there exists two groups, $G_1(v)$ and $G_2(v)$, the first of which has mean valuations $\bar{v}_A^1$ and $\bar{v}_B^1$ and the second of which has mean valuations $\bar{v}_A^2$ and $\bar{v}_B^2$. Individually, these groups’ mean buyer surplus is $\min\{\bar{v}_A^1, \bar{v}_B^1\} - c$ and $\min\{\bar{v}_A^2, \bar{v}_B^2\} - c$. If the groups merge their mean valuations are $\bar{v}_A^{12} = \alpha \bar{v}_A^1 + (1-\alpha)\bar{v}_A^2$ and $\bar{v}_B^{12} = \alpha \bar{v}_B^1 + (1-\alpha)\bar{v}_B^2$. 


where \( \alpha = \int dG^1(\psi) / \int dG^1(\psi) + G^2(\psi) \) is the fraction of the merged group who were members of Group 1. The combined group's mean buyer surplus is \( \min[v^1_{A}, v^2_{B}] - c \). So the merger increases total buyer surplus if

\[
\min[v^1_{A}, v^2_{B}] - c \geq \alpha \min[v^1_{A}, v^1_{A}] + (1 - \alpha) \min[v^2_{A}, v^2_{B}] - c.
\]

(8)

Suppose \( v^1_{A} > v^1_{B} \) and \( v^2_{A} > v^2_{B} \) so both groups purchase from Firm A if they don't merge (a similar argument holds when \( v^1_{B} > v^1_{A} \) and \( v^2_{B} > v^2_{A} \)). Then the merged group purchases from Firm A and its mean valuation is for Product B is \( v^1_{B} = \alpha v^1_{A} + (1 - \alpha)v^2_{B} \). So (8) holds with strict equality.

Suppose instead that \( v^1_{A} > v^1_{B} \) and \( v^2_{A} > v^2_{B} \), so Group 1 purchases from Firm A and Group 2 purchases from Firm B if they don't merge (a similar argument holds when \( v^1_{B} > v^1_{A} \) and \( v^2_{B} > v^2_{A} \)). Then the merged group's mean buyer surplus if it buys from Firm A is

\[
v^1_{B} - c = \alpha v^1_{B} + (1 - \alpha)v^2_{B} - c > \alpha v^1_{A} + (1 - \alpha)v^2_{A} - c,
\]

and the merged group's mean buyer surplus if it buys from Firm B is

\[
v^2_{A} - c = \alpha v^1_{A} + (1 - \alpha)v^2_{A} - c > \alpha v^1_{B} + (1 - \alpha)v^2_{B} - c,
\]

so (8) holds with strict inequality. \( \square \)

**Proof of Proposition 2.** First consider \( \psi_A > \psi_B \) (a similar argument holds when \( \psi_A < \psi_B \)). If a CP-SPNE with buyer groups exists, then by Lemma 4, \( v^i_{A} > v^i_{B}, \forall i \) and all buyers with valuations \( v_B > v_A \) must join groups, and by Lemma 3, every buyer’s equilibrium surplus is \( v_B - c \).

When all groups satisfy \( v^i_{A} > v^i_{B} \) and all buyers with valuations \( v_B > v_A \) are in groups, then every buyer group pays a price equal to \( v^i_{A} - v^i_{B} + c \) and every buyer group buys Product A (or mixes between Product A and Product B).

If \( v^i_{A} > v^i_{B} \) for some \( i \) then each buyer in \( i \) has surplus equal to \( v_A + t^i(v_A, v_B) - (v^i_{A} - v^i_{B}) \). So the transfer payments, \( t^i(v_A, v_B) = v_A - v_B - (v^i_{A} - v^i_{B}) \), are feasible and imply that every buyers’ surplus is \( v_B - c \), so joining the group is clearly individually rational.

If \( v^i_{A} = v^i_{B} \) then each buyer in \( i \) has surplus equal to \( (v_A + v_B)/2 + t^i(v_A, v_B) - c \), and the transfer payments, \( t^i(v_A, v_B) = 0 \), are feasible and imply that every buyers’ surplus is \( v_B - c \), so joining the group is clearly individually rational.

Finally, since the equilibrium payoffs are higher than the deviation payoffs for any potential group \( G^i(\psi) \),

\[
\frac{f(v_B - c)G^1(\psi)}{\int G^i(\psi)dv} = v^i_{B} - c \geq \min[v^1_{A}, v^1_{B}] - c, \quad \forall G^i(\psi),
\]

it follows that the groups are coalition proof, so the groups represent a CP-SPNE.

Next, consider \( \psi_A = \psi_B \). If the groups are a CP-SPNE, then by Lemma 4, \( v^i_{A} = v^i_{B} \), \( \forall i \), and all buyers must join groups, and by Lemma 3, each group’s average surplus is \( (v^i_{A} + v^i_{B})/2 - c \). However, the individual buyer’s surplus is not uniquely defined. By Lemma 3, \( t^i(v_A, v_B) = \theta(v_A - v_B) \). Also, note that in any equilibrium \( \theta \) must be the same across buyer groups, otherwise buyers in low \( \theta \) groups for whom \( v_A > v_B \) would be strictly better off forming a new coalition with buyers in high \( \theta \) groups for whom \( v_B > v_A \). So each individual buyer’s surplus is \( (v_A + v_B)/2 + \theta(v_A - v_B) - c = (p v_A + (1 - p)v_B) - c \), where \( p = 1/2 - \theta \) and \( \theta \in [-1/2, +1/2] \), so \( p \in [0, 1] \).

Also, when all groups satisfy \( v^i_{A} = v^i_{B} \) and all buyers are in buyer groups, then each buyer’s surplus is \( (v_A + v_B)/2 + t^i(v_A, v_B) - c \), and the transfer payments, \( t^i(v_A, v_B) = 0 \), are feasible and imply that every buyers’ surplus is \( v_B - c \), so joining the group is clearly individually rational and coalition proof.

Finally, if buyers’ preferences are unanimous, by Lemma 3 the only buyer groups that can form are trivial since the equilibrium payoffs and consumption is identical to the payoff and consumption when all buyers are independent. \( \square \)

**Proof of Lemma 5.** I begin by proving that the groups defined by the price vectors form a CP-SPNE. First, I show that there existing transfer payments satisfying budget balance that are associated with the groups defined by the prices, and then that these groups are individually rational and coalition proof. If \( \psi_j = \psi_k^j \) then in equilibrium the firms’ price offers are equal to cost, the buyer group mixes between purchasing product \( s \) and product \( t \), and the transfer payments, \( t^j_k(\psi) = p^j_kv_j + (1 - p^j_k)v_k - v_j + v_k)/2 \) are clearly feasible, and \( \int t^j_k(\psi)dv = 0 \), and imply that each buyer’s surplus is \( p^j_kv_j + (1 - p^j_k)v_k - c \).

If \( p^j_k = 0 \) and \( \psi_j < \psi_k^j \) then in equilibrium the buyer group purchases product \( t \) at a mean price of \( (\psi_k^j - \psi_j) + c \), and the transfer payments, \( t^j_k(\psi) = (v_j - v_k) + (\psi_k^j - \psi^j_k) \) are clearly feasible, and \( \int t^j_k(\psi)dv = 0 \), and imply that each buyer’s surplus is \( v_j - c \), or equivalently, \( p^j_kv_j + (1 - p^j_k)v_k - c \).
If \( p_{jk} = 1 \) and \( \bar{v}^{ik}_{jk} > \bar{v}^{ik}_{k} \), then in equilibrium the buyer group purchases product \( s \) at a mean price of \((\bar{v}^{i} - \bar{v}^{ik}_{jk}) + c\), and the transfer payments, \( t^{jk}(\mathbf{v}) = (v_j - v_j) + (\bar{v}^{jk}_{jk} - \bar{v}^{jk}_{j}) \) are clearly feasible, since \( \int t^{jk}(\mathbf{v}) dG^{jk}(\mathbf{v}) = 0 \), and imply that each buyer’s surplus is \( v_j - c \), or equivalently, \( p_{jk} v_j + (1 - p_{jk}) v_k - c \).

Furthermore, each buyer’s decision is clearly individual rational because a type \( j > k \) can choose to join a type \( jk \) group, which weakly dominates purchasing independently. Now suppose that the groups implied by the prices are not coalition proof. Specifically, suppose a new group exists such that surplus for any potential group satisfying it follows that \( p_{jk} v_j + (1 - p_{jk}) v_k - c \). The aggregate surplus for any group satisfying \( \bar{v}^{i}_{j} = \bar{v}^{i}_{k} \) is exactly \( \bar{v}^{i}_{k} - c \), so this implies \( \bar{v}^{i}_{k} + p_{jk}(\bar{v}^{i}_{j} - \bar{v}^{i}_{k}) - c > \bar{v}^{i}_{k} - c \), which is a contradiction. Similarly, the aggregate surplus for any potential group satisfying \( \bar{v}^{i}_{j} > \bar{v}^{i}_{k} \) is \( \bar{v}^{i}_{k} + p_{jk}(\bar{v}^{i}_{j} - \bar{v}^{i}_{k}) - c > \bar{v}^{i}_{j} - c \) which is also a contradiction. Finally, a similar argument holds for groups satisfying \( \bar{v}^{i}_{j} < \bar{v}^{i}_{k} \). So these buyer groups and transfer prices form a CP-SPNE.

Next I show there is a price vector that generates every CP-SPNE. More precisely, I prove that every CP-SPNE is payoff equivalent to a CP-SPNE that is defined by a vector of prices.

Suppose not. Because every CP-SPNE is payoff equivalent to a CP-SPNE with at most \( n(n-1)/2 \) groups, for one of every pair of firms, this means that there exists a CP-SPNE, and set of groups \( G(\mathbf{v}) \) and transfer payments \( t(\mathbf{v}) \) that is feasible, individually rational, and coalition proof, but \( G \) cannot be derived from a price vector \( \mathbf{p} \). By Lemma 3 (which clearly applies when \( n > 2 \) firms as well when \( n = 2 \)) the surplus of each individual buyer can be written as \((1/2 + \theta \mathbf{v}) v_j + (1/2 - \theta \mathbf{v}) v_j \) where \( \theta \mathbf{v} \) is unique to each group.

Let \( \mathbf{p}^{ij} = 1/2 + \theta \mathbf{v} \). By assumption, these prices to not yield the groups \( G(\mathbf{v}) \). This means that for some \( i \) and some \( j \), some of the type \( i \) consumer is not in group \( ij \) but would earn a strictly higher surplus if the were. But in this case some \( j > i \) consumers in group \( ij \) would be strictly better off forming a new coalition with these consumers. So \( G \) must be defined by \( \mathbf{p}^{ij} \).

**Proof of Proposition 8.** From (1), let \( C^{jk}(\mathbf{v}; \mathbf{p}) \) denote the distribution of buyers that choose to join group \( jk \) given the price vector \( \mathbf{p} \). Clearly \( C^{jk}(\mathbf{v}; \mathbf{p}) \) is single valued and continuous in \( \mathbf{p} \) for all \( \mathbf{p} \in (0,1)^{n(n-1)/2} \). For all \( \mathbf{p} \in (0,1)^{n(n-1)/2} \), define the net supply of buyers with valuations \( v_j > v_k \) buyers to group \( jk \) to be

\[
\mathbf{z}^{jk}(\mathbf{p}) = \int (v_j - v_k) dG^{jk}(\mathbf{v}; \mathbf{p}) = \bar{v}^{i}_{j} - \bar{v}^{i}_{k}.
\]  

Clearly \( \mathbf{z}(\mathbf{p}) \) is a bounded and continuous function on the open set \((0,1)^{n(n-1)/2}\). Given a point \( \mathbf{p}' \) on the boundary of \((0,1)^{n(n-1)/2}\) and any sequence of prices in \((0,1)^{n(n-1)/2}\) that converge to \( \mathbf{p}' \), define \( \mathbf{z}(\mathbf{p}') \) to be the limit \( \lim_{\mathbf{p} \rightarrow \mathbf{p}'} \mathbf{z}(\mathbf{p}) \). Given this, it follows that \( \mathbf{z}(\mathbf{p}) \) is a bounded and continuous function on \((0,1)^{n(n-1)/2}\).

Now define a mapping from \([0,1)^{n(n-1)/2}\) to \([0,1)^{n(n-1)/2}\) as follows

\[
f(\mathbf{p}) = \{ \mathbf{q} \in [0,1)^{n(n-1)/2} : \mathbf{q}^{ij}(\mathbf{p}) = \mathbf{q}'^{ij}(\mathbf{p}) \text{ for all } \mathbf{q}' \in [0,1)^{n(n-1)/2} \}.
\]  

Since \( \mathbf{z}(\mathbf{p}) \) is continuous, it follows that \( f(\mathbf{p}) \) is non-empty, convex-valued, and has a closed graph. And by Kakutani’s fixed point theorem \( f(\mathbf{p}) \) must have a fixed point. Note that \( \mathbf{p}^{ij} = f^{jk}(\mathbf{p}) = 1 \) whenever \( \mathbf{z}^{jk}(\mathbf{p}) < 0 \) and \( \mathbf{p}^{ij} = f^{jk}(\mathbf{p}) = 0 \) whenever \( \mathbf{z}^{jk}(\mathbf{p}) > 0 \), so conditions (i), (ii), and (iii) of Lemma 5 are satisfied. So the fixed point of \( f(\mathbf{p}) \) must be a price vector that defines a set of buyer groups which constitutes a CP-SPNE.

**Proof of Proposition 9.** First, suppose \( p_{ij} = p_{jk} = 1 \) and \( p_{ik} = 0 \). Then buyers sort into three groups and all buyers with the same highest valuation join the same group. This implies \( \bar{v}^{ij}_{i} < \bar{v}^{ij}_{j} \) (and a similar condition for each of the other groups) which is a contradiction of condition (ii).

Similarly, suppose \( p_{ij} = p_{jk} = p_{ik} = 1 \). Then all buyers whose highest valuation is \( v_i \) join one of two groups, \( ij \) or \( ik \), and all buyers whose highest valuation is \( v_j \) join group \( jk \), and all buyers whose highest valuation is \( v_k \) join one of two groups, \( jk \) or \( ik \). But \( \bar{v}^{ik}_{jk} \leq \bar{v}^{ik}_{k} \) only if all buyers whose highest valuation is \( v_k \) join \( jk \). Otherwise condition (ii) is violated. But if this is the case, then it follows that the only members of both \( ij \) and \( ik \) are buyers whose highest valuation is \( v_i \), so \( \bar{v}^{ij}_{i} > \bar{v}^{ij}_{j} \) or \( \bar{v}^{ik}_{i} > \bar{v}^{ik}_{k} \) or both, which is a contradiction of condition (ii).

More generally, if all three prices are either \( 0 \) or \( 1 \), then an argument analogous to one of the above arguments holds and either condition (ii) or condition (iii) is violated.

Second, suppose \( p_{ij} = 1, p_{ik} = 0 \) and \( 0 < p_{jk} < 1 \). Then all buyers whose highest valuation is \( v_i \) join \( ij \) and all buyers whose highest valuation is \( v_j \) join \( ik \) and all buyers whose highest valuation is \( v_k \) and second highest valuation \( v_j \) join \( jk \). This implies that the only other buyers to join \( ijk \) buyers whose highest valuation is \( v_j \) and second highest valuation is \( v_i \) (and not all of these). Since group \( ij \) contains all buyers whose highest valuation is \( v_i \), clearly \( \bar{v}^{ij}_{i} > \bar{v}^{ij}_{j} \) which is a contradiction of condition (ii). A similar contradiction arises for group \( ik \).
Similarly, suppose \( p_{ij} = p_{ik} = 1 \) and \( 0 < p_{jk} < 1 \). Then all buyers whose highest valuation is \( v_j \) join \( ij \) or \( ik \) and all buyers whose highest two valuations are \( v_j \) and \( v_k \) join \( jk \). Some of the buyers whose highest valuation either \( v_j \) or \( v_k \) and second highest valuation is \( v_i \) will join either \( ij \) or \( ik \) while others will join \( jk \). However, pooling the buyers that join \( ij \) and \( ik \), since the pooled group contains all the buyers whose highest valuation is \( v_j \) (and some whose second highest valuation is \( v_i \) and third highest valuation is \( v_j \)) and only some of the buyers whose highest valuation is \( v_j \) and second highest valuation is \( v_i \) (and none of the buyers whose second highest valuation is \( v_j \) and third highest is \( v_i \)), so clearly \( v_{ij}^{p_{ij}} > v_{ik}^{p_{ik}} \) which is a contradiction of condition (ii).

More generally, if exactly two prices are either 0 or 1, an argument analogous to one of the two above arguments holds and either condition (ii) or condition (iii) is violated.

Finally suppose that at most one price is 0 or 1, but the prices are not equal. With loss of generality assume \( p_{12} > p_{13} > p_{23} > 0 \) and assume \( p_{12} < 1 \) but that all other prices are in the open interval \((0, 1)\). Then it follows that the following must hold:

\[
E[v_1|p_{12}v_1 + (1 - p_{12})v_2 \geq \max\{p_{13}v_1 + (1 - p_{13})v_3, p_{23}v_2 + (1 - p_{23})v_3\}]
\geq E[v_2|p_{12}v_1 + (1 - p_{12})v_2 \geq \max\{p_{13}v_1 + (1 - p_{13})v_3, p_{23}v_2 + (1 - p_{23})v_3\}],
\]

(12)

\[
E[v_2|p_{23}v_2 + (1 - p_{23})v_3 \geq \max\{p_{13}v_1 + (1 - p_{13})v_3, p_{12}v_1 + (1 - p_{12})v_2\}]
= E[v_3|p_{13}v_1 + (1 - p_{13})v_3 \geq \max\{p_{13}v_1 + (1 - p_{13})v_3, p_{12}v_1 + (1 - p_{12})v_2\}]
\]

(13)

\[
E[v_1|p_{13}v_1 + (1 - p_{13})v_3 \geq \max\{p_{12}v_1 + (1 - p_{12})v_2, p_{23}v_2 + (1 - p_{23})v_3\}]
= E[v_3|p_{13}v_1 + (1 - p_{13})v_3 \geq \max\{p_{12}v_1 + (1 - p_{12})v_2, p_{23}v_2 + (1 - p_{23})v_3\}].
\]

(14)

Note that there are three (or \( n(n-1)/2 \)) potential groups, \([12, 13, 23]\), and six (or \( n(n-1) \)) categories of buyers, \([1 > 2 > 3 > 2, 2 > 1 > 3, 2 > 3 > 1, 3 > 1 > 2, & 3 > 2 > 1]\).

Clearly the buyers for whom \( 1 > 2 > 3 \) all choose to be in 12. Group 12 puts greater weight on \( v_1 \) and their preferences satisfy \( v_2 > v_3 \), so 13 is not attractive (and 23 never is to buyers who value good 1 the highest).

It also follows that buyers for whom \( 3 > 2 > 1 \) will all choose to be in 23. Group 23 puts greater weight on \( v_3 \) and \( v_2 > v_1 \), so 13 is not attractive (and 12 never is).

But some buyers for whom \( 1 > 3 > 2 \) will choose 12 instead of 13 because the weight on \( v_1 \) is higher in 12 (i.e., \( p_{12} > p_{13} \)). This will be a strictly positive measure if \( p_{12} > p_{13} \) given that \( F \) is strictly increasing. Also, some buyers for whom \( 3 > 1 > 2 \) may choose 23 instead of 13 because the weight on \( v_3 \) is higher in 23 (i.e., \( 1 - p_{23} > 1 - p_{12} \)). In fact, this will be a strictly positive measure if \( 1 - p_{23} > 1 - p_{12} \), given the assumption that \( F \) is strictly increasing. So group 12 is attracting some extra buyers, beyond the \( 1 > 2 \) buyers, who prefer good 1 to good 2, and group 23 is attracting extra buyers, beyond the \( 3 > 2 \) buyers, who prefer good 3 to good 2.

A necessary condition for the first weak inequality to hold is that some additional buyers, beyond the \( 2 > 1 > 3 \) buyers, choose to join group 12. Assuming group 12 attracts all of the \( 2 > 1 > 3 \) buyers, it still must still attract some \( 2 > 3 > 1 \) buyers to choose 12 instead of 23 or the first equality is clearly violated. This will happen only if the weight on \( v_2 \) is higher in 12 then in 23, or \( 1 - p_{12} > p_{23} \).

And a necessary condition for the third equality to hold is that some other additional buyers, beyond the \( 2 > 3 > 1 \) buyers, choose to join group 23. Assuming group 23 gets all the \( 2 > 3 > 1 \) buyers, it still must attract some \( 2 > 3 > 1 \) buyers to choose 23 instead of 12 or the third equality is clearly violated. This will happen only if the weight on \( v_2 \) is higher in 23 than in 12, or \( p_{23} > 1 - p_{12} \). But this is a contradiction.

Finally, if \( p_{12} = p_{13} = p_{23} \), then clearly \( p_{12} = p_{13} = p_{23} = 1/2 \). If \( p_{12} = p_{13} = p_{23} < 1/2 \) then all \( 2 > 3 > 1 \) buyers and some \( 2 > 1 > 3 \) buyers join group 23 but only the \( 3 > 2 > 1 \) buyers, and not the \( 3 > 2 > 2 \) buyers, join group 23, so the second condition is violated. Similarly, if \( p_{12} = p_{13} = p_{23} > 1/2 \) then all \( 2 > 1 > 3 \) buyers and some \( 2 > 3 > 1 \) buyers join group 12 but only the \( 1 > 2 > 3 \) buyers, and not the \( 1 > 3 > 2 \) buyers, join group 23, so the first condition is violated.

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References


